

Combining Share and Measure Meaning of Fractions to Facilitate Students' Reasoning

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It is well documented that one of the most difficult topics that children learn at the upper primary and lower middle school level is 'fractions'. School curricula in India often introduce fractions in class III and by class V children are expected to learn all the operations on fractions. Yet it would not be uncommon to find children in class VII who would say $1/3$ is bigger than $1/2$ and $1/2 + 1/4 = 2/6$. As most textbooks use an approach that is basically algorithmic, children learn about fractions in a mechanical way without any conceptual grasp. Moreover the textbooks exclusively focus on the part-whole meaning of fractions. The part-whole meaning and the associated vocabulary is restrictive in that it does not easily facilitate reasoning in the context of tasks of comparing fractions or finding equivalent fractions (However, for a different treatment of the part-whole relation see Lamon, 2002)

In this paper, we report data from trials with four different groups of students using an approach that combines the share and the measure interpretation of fractions. We will present examples of students' reasoning that indicate that teaching fractions using a combination of sharing and measure interpretations might prove to be a better approach in facilitating students' reasoning about fractions. Though fractions can be understood through various sub-constructs (Charalambous and Pitta-Pantazi, 2007), we emphasise here the two sub-constructs of sharing and measure and interrelation between these.

The share interpretation:

The equal share interpretation (abbreviated as 'share interpretation') defines a fraction m/n as the share one gets when m things are divided equally over n children and with this definition arrives at relations between fractions by logical reasoning (Streefland, 1991). For example, different ways in which 5 things can be shared equally among 6 children gives us the following relations:

If each of the five objects is shared equally among 6 children we get the relation

$$5/6 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 5 \times (1/6)$$

If we share one object among two, each of the 6 children would get $1/2$ share of the object and we would use up three of the five objects in this process. From the remaining if we share one object among three, each of the six children would get a $1/3$ share more. This gives us the relation

$$5/6 = 1/2 + 1/3$$

As there are only five things to be shared equally among 6 children, it is clear that the share of each child, that is $5/6$, is less than 1. By dividing each object into 6 equal parts and removing one part and distributing the remaining to one child, five children of the six children and finally distributing the 5 parts we removed to the sixth child, we get the relation

$$5/6 = 1 - (1/6)$$

One advantage of the share interpretation is that it enables us to deal with proper fractions and improper fractions simultaneously and converting an improper fraction into mixed fraction becomes automatic. For example, $5/4$ is the share one child gets when 5 objects are shared equally among 4 children and if we first distribute one object each to each of the four children, then there is one object remaining, which when divided equally among 4 would give a share of another $1/4$ to each. So we get the relation, $5/4 = 1\frac{1}{4}$. Also, one can work with large numbers almost from the beginning. To see that $42/57$ is less than $45/54$ is easy because we have 3 objects less than 45 and 3 children more than 54. When fewer things are shared among larger number of children, the share would naturally be less. Children seem to relate to the share meaning in an intuitive way, though initially they have an incomplete understanding: $5/6$ for them denotes five things to be shared equally among 6 children and not the share of each child. The share

interpretation is also the division or quotient interpretation of fraction, which is important for developing students' ability to solve problems involving multiplicative and linear functional relations (Subramaniam and Naik, 2007).

The measure interpretation:

One question that the share interpretation does not answer is 'how much' is a certain given amount. To do this we need the measure interpretation. The measure interpretation defines first the unit fraction $1/n$ as the measure of one part when the whole is divided into n equal parts and then defines the composite fraction m/n as the measure of m such $1/n$ parts. So the fraction $5/7$ is made of 5 piece units of size $1/7$ and the fraction $7/5$ is made of 7 piece units of size $1/5$. Also 5 piece units of size $1/5$ make 1 and 6 piece units of size $1/6$ make 1 or in general n piece units of size $1/n$ make 1. So it follows that

$$7/5=1+2/5$$

Even though the interpretation of fraction as a measure involves partitioning of the whole into equal parts, the emphasis on each part as the (sub)unit makes it different from the part whole interpretation of fraction.

A question that arises in this context is whether children are able to meaningfully integrate the share, measure and the part-whole interpretation that they may know from previous instruction or whether they experience a conflict between these different interpretations. In this paper, we report data from trials conducted with four groups of students that seeks to answer this question.

Our trials:

We report data from trials that were conducted with four different groups of children between the ages 10 to 12 years, taught by the authors. Three groups of children study in formal schools located in different cities in India in three different languages of instruction. The fourth group of children study at an informal school run by an NGO working with rag pickers. The trials used an approach that combined the measure and the share interpretations, but had slight variations across the different groups. In the paper we describe the tasks used in the trials and analyze the varied responses of the children to the tasks. We found children using their own arguments and ways of reasoning reflecting a strong understanding of fractions. One of the findings was that children used both the share and the measure meaning in their reasoning suggesting that these mutually reinforce each other. We argue that share interpretation combined with the measure interpretation might prove to be a better approach for teaching fractions.

References

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