Attending to Language, Culture and Children's thinking as they Learn Fractions

K. Subramaniam and Shweta Naik
Homi Bhabha Centre for Science Education
Tata Institute of Fundamental Research, Mumbai , India

The fraction notation and the arithmetic of fractions consolidate and extend the ability to represent and manipulate multiplicative relations. Thus they provide the tools to deal with the full range of situations involving proportionality and also prepare the student for algebra. However, fractions are known to be a difficult topic for most students. One of the sources of difficulty is the fact that when fractions are applied to situations, they encode a variety of meanings whose unity is not apparent: measure, quotient, ratio and operator (Kieren, 1993; Charalambous and Pitta-Pantazzi, 2007). A part of the difficulty is because the fraction notation symbolizes division, and the division operation and the associated procedures include some of the most difficult concepts and routines that students encounter in the early grades. Understanding a fraction as a number creates notational and conceptual difficulties for students accustomed to whole numbers (Vamvakoussi and Vosniadou, 2004). Further, fractions in general form (that is, of the form m/n) do not find support in the culture either in the form of material artefacts or language. At best we find support in the culture for the binary ($1/2^n$) (Palhares, this Research Forum) or the decimal ($1/10^n$) fractions.

Researchers have devised and studied many innovative approaches including tasks, contexts and manipulatives that are meaningful to children and help in learning some sub-concept related to fractions (See for example, Streefland, 1996; Lamon, 2002; Steencken and Maher, 2003). Some of the tasks and situations, such as the equal sharing situation (Streefland), resemble those that children encounter in real life, while some tasks like the 'varying units' task (Lamon, Steencken and Maher), do not resemble real life situations.

Parallel to the strand of research on the teaching and learning of fractions, there have been cognitive and cultural-anthropological studies of everyday mathematical learning (Carraher et al., 1985; Khan, 2004). These studies have shown that there are possible sources of learning from outside the school context that can potentially help in learning school mathematics, especially in relation to multiplicative thinking and fractions. The use of measurement and proportionality is ubiquitous in real life, and occurs in contexts of trade, small-scale production and household activities. All these contexts are likely to be familiar to many children, especially those from poor, urban neighbourhoods in the developing world. Since these contexts abound in multiplicative relationships, connecting them with what children learn about fractions in school would serve to deepen children's interest and knowledge and would make it more likely that they experience education as authentic and empowering. The mathematical facts, concepts and tools that children learn from out-of-school contexts are different from those that they learn in school, although the two are related (See Nunes, this Research Forum). The broad question that forms the background of this presentation is, 'how can these two be related in the work of teaching and learning in a way that facilitates student learning in the domain of fractions?'

Developing a framework that illuminates the relation between out-of-school mathematics and school mathematics in detail is an essential step in using students' out-of-school knowledge in a powerful manner. Often teachers are aware of what students know or can do, and may use this information sporadically while explaining concepts, but are unable to relate students' out-of-school knowledge in a rich and meaningful manner with what is being learnt in school. Appreciating students' thinking, attending to the language they use, and to the culture they are immersed in, is possible when teachers are aware of the core differences as well as the potential points of connection between school mathematics and the culture.

One of the core differences between everyday mathematics and school mathematics is that the latter includes the arithmetic of fractions. This arithmetic entails that fractions be treated as numbers in their own right, and not only as ratios or quotients of whole numbers. Thus students need to understand how fractions (restricted to positive rational numbers) designate quantities, which in turn, requires the understanding of unit fractions. The interpretation of fraction as a number is often described as the 'measure' interpretation, and involves positioning a fraction on the number line. Fraction as measure includes the following ideas (i) a fraction denotes a quantity, (hence a point on the number line) (ii) arithmetic operations are possible with fractions like with other numbers (ii) each operation models certain situations and transformations in the world. While everyday culture may facilitate the construction of the measure meaning of a few fractions such as half or quarter, constructing the measure meaning of fractions in general m/n form needs the more formal setting of school mathematics. The question that we wish to raise for discussion is how language and culture are a source of constraints and affordances in constructing the measure meaning of fractions.

Studies have repeatedly found that young children can understand, and can often carry out the equal division of continuous wholes or discrete collections (Confrey, ICME 11 presentation, 2008). Some researchers have suggested that along with counting, the equal partitioning scheme is basic and develops parallel to and independently of the counting scheme. The equal partioning scheme is the foundation for understanding both whole number division and fractions. It has also been shown that 2-splits and recursive 2-splits (i.e., the binary or 2^n-splits) are easier for young children than odd-splits and 'composite number splits' (Hiebert and Tonnessen, 1978). Children often exploit the symmetry in continuous wholes to carry out these splits. It is likely that culture and cognition mutually reinforce the equal partitioning scheme and strengthen the bias for binary partitions, given the ubiquitous use of 'half', the frequent appearance of 'quarter', and the occasional appearance of smaller binary fractions in various cultures. Extending the 2-split scheme to the n-split scheme involves the co-ordination of spatial and quantitative understanding (Confrey, *op cit.*), as for example, when children learn to divide a circular shape into three equal parts. Studying how the n-split scheme is elaborated, and more importantly, how children disambiguate shape congruence and equality of quantity is worthy of study. Clearly language and context would play an important part here.

Moving from equal partitioning to using the result of partitioning as a unit of measure is an important step. When the unit fractions are available as units, composite fractions can be interpreted as measures, as multiples of unit fractions. This is a different interpretation from the part-whole interpretation of composite fractions (Naik and Subramaniam, 2008). With respect to the fractions half and quarter, the culture already prepares this transition since the words for half and quarter go beyond merely recording the act or the result of equal division. They also designate measures and operators. There are numerous contexts in which 'half' is used to communicate a measure and is readily understood: half litre, half cup, half packets, etc. Similarly one can speak about the operations of halving, reducing by half, etc. As a measure, half is sometimes combined with other units: the languages in the North of India contain special words for 1½ and 2½. Further, we also find composite fractions such as 'three-quarters' – there are words in common usage in most Indian languages for ¾.

Thus one of the transitions that children need to make is to treat the units obtained from *'n'* equal partitions as new units of measure. Unit fractions form a pivotal concept but everyday culture and language may provide minimal or no support for this concept. Even the choice of fraction words in school mathematics may or may not highlight unit fractions as units. In school textbooks in the Indian languages, for example, unit fractions are not designated by ordinal number words (one-third, one-fourth, etc.). In the languages of North India, unit fractions as well as composite fractions are named by words that indicate division – the fraction 1/5 is read as 'e*k bate paanch*' in Hindi, which means 'one divided or shared among five'. In the languages of South India, unit as well as composite fractions are named by words conveying a part-whole meaning – the suggested reading of the fraction 2/5 in Tamil school textbooks is '*ainthil erandu*' meaning 'two out of five' or 'two parts out of five'. Even when students study in the English language, the commonly used words for fractions are simply taken from their Indian language equivalents. Thus it is far more common to hear teachers and students in English medium schools read 2/5 as 'two by five' or 'two out of five' rather than as 'two-fifths', even though the ordinal forms appear in some of the school textbooks. Such language imposes constraints on the strands of meaning that children can easily access while learning fractions. For example, indicating composite fractions as multiples of unit fractions (captured in the word 'two-fifths') is awkward using the part-whole language. Saying 'two of one part out of five', is needlessly complex in comparison to 'two parts out of five', but the latter phrase tends to reinforce the part-whole sense.

The division form used in Indian schools for composite fraction words (e.g. 'two by five') suggests a strong connection with the equal sharing situation. Such situations are familiar to children, have counterparts in the culture, and have been used effectively in building the beginning understanding of fractions (Streefland, 1996). They also provide an opportunity for the elaboration of equal partitioning strategies. However, without independent access to the measure interpretation, the equal sharing situations tend to reinforce a ratio or quotient interpretation of fractions – m/n is 'm units shared among n children' rather than a representation of the quantity of each share – an interpretation which still remains in the whole number domain (Naik and Subramaniam, 2008). One reason may be that in most everyday situations, the goal of equal sharing does not really need a further quantification of the share beyond establishing that the shares are equal. The need to quantify may arise when the share is used for some other purpose. For example, when a quantity of raw material for a recipe is shared, one may need to determine whether the share is sufficient to make the final product.

While equal sharing situations rely wholly on the equal partitioning scheme, proportion situations embed equal partitioning within a logic of correspondences. Proportion problems are solved in out-of-school contexts and sometimes by children in school, by carrying out corresponding operations of partitioning and replicating in the two measure spaces (Nunes, this Research Forum). Hence in these contexts both quantification and partitioning actions are important. The fraction notation allows one to represent the scalar operation of partitioning and replicating within a measure space as a single multiplier (Behr et al.,). One needs to study whether this approach is more consistent with the informal strategies of everyday mathematics, than an approach using the rule of three, or a unit-rate approach. Some theoretical studies (Shwartz, 1988) have suggested that multiplication is most often a referent changing operation (rate measure of type 1 = measure of type 2; candies per bag number of bags = number of candies), which makes multiplicative structures different from and conceptually more difficult than additive structures. We see in many of the students' and adults' informal responses (Nunes, this Research Forum) an avoidance of referent changing operations. Proportion problems are solved by using operations within a measure space rather than operations that connect two measure spaces. Hence interpreting fractions as operators within a measure space may be closer to the informal strategies. Further, partitioning a measure may yield a measure that can only be represented using fractions. Thus proportion situations bring partitioning together with the need to represent operators and measures, and hence furnish contexts where fractions can be meaningfully applied.

Cultural contexts are rich in a variety of units of measure that arise due to convenience. We have found whole number units of ten, dozen, twenty and larger occuring in small-scale household based production activities, such as packaging, assembling, etc. Problems of the interconversion of units, which are a variety of proportion problems, are frequently encountered and solved in such contexts. Usually denoting measures in terms of fractions is avoided, and the availability of sub-units allows one to work with whole numbers. (Rather than one-fifth packet, it is common to refer to 80 sheets, since a packet of paper contains 400 sheets.) Nevertheless, the fact that children know and deal with so many different units in these situations suggests that it is possible to develop fractions as measures in such situations.

Although there is a lack of support for the semantics of the fraction notation through culture, we see that culture can provide approaches to the understanding fractions through equal sharing, ratio and proportion situations. Such situations make use of partitioning schemes and multiplicative structures leading to various meanings of fractions as quotient, as measure or as operator. Examining informal strategies by children and adults in out-of-school contexts reveals new possibilities for a trajectory towards meaningful learning of fractions.

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