Introduction

Many students in our schools even today show displeasure towards the learning of mathematics. Students view mathematics as dull, boring, and stereotyped. Most students think that mathematics means to ‘get something right’ or to ‘get something wrong’. When they get it wrong they think that they are not good enough for mathematics and lose interest in learning. On the other hand a few students learn enough tricks to ‘get something right’ to pass the exam.

There are not enough instances when a teacher has tried to teach maths in a different way, say through activities and games. These activities and games make the process of learning more interesting and effective. The setting up of a mathematics laboratory aids this effort.

A mathematics laboratory is a place where we find a collection of games, puzzles, teaching aids and other materials for carrying out activities. These are meant to be used both by the student by their own and together with their teacher to explore the world of mathematics, to discover, to learn and to develop an interest in mathematics. Although mathematics is not an experimental science in the way in which physics, chemistry and biology are, a mathematics laboratory can contribute greatly to the learning of mathematical concepts and skills.

Here are some ways we think a mathematics laboratory could contribute to learning mathematics:

- A mathematics laboratory provides an opportunity for the students to discover through doing. In many of the activities, students learn to deal with problems while doing concrete activity, which lays down a base for more abstract thinking.
- It gives more scope for individual participation. It encourages students to become autonomous learners and allows a student to learn at his or her own space.
- It widens the experiential base, and prepares the ground for later learning of new areas in mathematics and of making appropriate connections.
- In various puzzles and games, the students learn the use of rules and constraints and have an opportunity to change these rules and constraints. In this process they become aware of the role that rules and constraints play in mathematical problems.
- Because of the larger time available individually to the student and opportunity to repeat an activity several times, students can revise and rethink the problem and solution. This helps to develop metacognitive abilities.
- It builds up interest and confidence in the students in learning and doing mathematics.
- Importantly, it allows variety in school mathematics learning.

In this report we have described some activities which could typically be included in a school mathematics laboratory. The activities are suitable for students of class 6 to class 10. We have also included a couple of activities suitable for a lower level — the place-value snake and the fraction chart. The items have been grouped under two broad headings: **activities** and **games and puzzles**. The activities could be done individually by students, with guidance from a teacher, or could be used for demonstration with a small group of students. Some of the activities could also be used as teaching aids in a classroom. The games and puzzles are fun to do individually and all of them contain some element of mathematics which can be explored while doing them or as a sequel. Some of these items have been developed by the authors at HBCSE. Others have been taken from articles and books and have been modified or developed further.

PART I

Activities

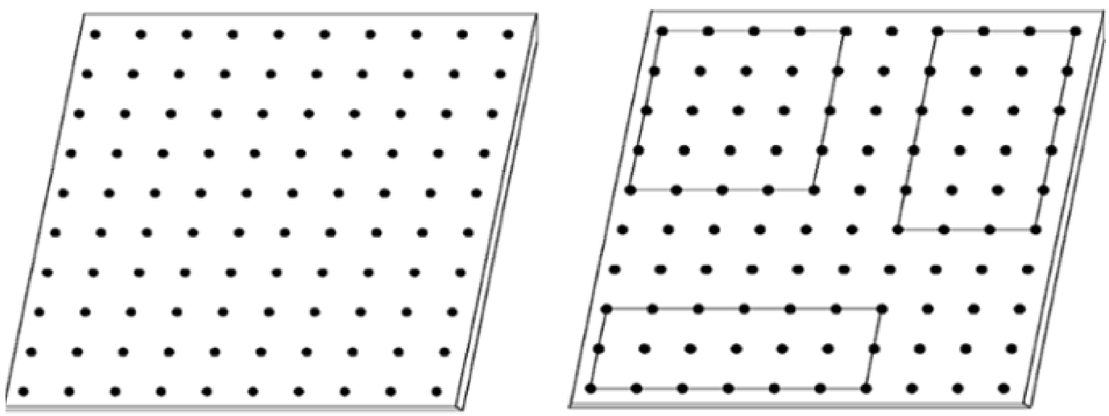
Area and Perimeter

Materials required: Square 10”×10” board of acrylic /plywood/ hardboard /chopped sole rubber, match sticks or short nails, string.

Many high school students find it difficult to master the concept of perimeter even though it is a simple concept. Further, its relation to area, which is the other measure associated with a bounded plane figure, is not clear. Some students even have difficulty with the concept of area. This activity clarifies the notion of perimeter and helps students discover its distances from the concept of area.

Drill holes on the square board at 1 inch intervals forming a square grid. The holes need to be just large enough to easily fit pegs which are about 2 mm in diameter. If the board is made of transparent acrylic sheet, it can also be used on the OHP. Another advantage with an acrylic sheet is that, it can be placed on tip of a graph paper, which can be used to estimate areas. (Materials such as thermocol, or chappal sole rubber can also be used if they are available easily.) Break the match sticks in half and scrape off the phosphorous for using them as pegs on the board. Alternatively short nails with small heads can be used. In all about a dozen pegs may be required.

PEG BOARD



The following activities can be performed with the pegboard:

- Take a length of string about a foot long and knot its two ends together to form a loop. Give the loop to students along with four pegs. What are the different kinds of figures which they can make? Ask them to name the figures. Ask them what remains the same as these figures change. Introduce the concept of perimeter as the total length of the boundary.
- Take a total string length of 20 inches which is knotted into a loop and four pegs. Ask the children to make rectangles of different sizes. Which rectangle has the largest area?
- With the same loop of string, ask the students to form figures with 3 pegs, with 5 pegs, 6 pegs and so on. Can they estimate the area? Which figure has the largest area?
- Ask students to construct figures with a given area and measure the perimeters. Which figure has the largest perimeter?

Geoboard

Materials required: Square board 10"× 10" of wood or acrylic, nails or pegs, superglue if acrylic board is used.

A geoboard is a very useful device for introducing children to important topics in school geometry. It consists of an array of nails or pegs which are placed at equal distances on a square acrylic board making up a grid. The baseboard is made from wood or acrylic. Drill holes at equal distances forming a square grid. It is convenient to place the pegs at gaps of 2 cm. Nails can also be used, but the heads must be small. Acrylic boards with holes drilled in them are good for use on an overhead projector. Graph paper can also be placed below an acrylic board which is useful in estimating areas. The pegs or nails can be glued onto the acrylic board.

Polygonal figures can be made on the geoboard by stretching rubber bands across the pegs.
A number of activities which are suitable for students of different classes can performed on the geoboard. Some of the activities that can be performed are

- Study of kinds of angles.
- Areas and perimeter
- Property of similar figures: Take two rubber bands and form the triangles one within the other. Are they similar? Why or why not?
- On the geoboard it is very easy to find areas of figures by completing rectangles. Hence it is easy to verify theorems about areas. For example, with two rubber bands the theorem, "the area of a triangle on the same base and between the same parallels is half that of the parallelogram" can be verified.
- Another activity on the geoboard is to construct right angle triangles such that none of the sides of the triangle is aligned along a row or a column. How many such triangles can be constructed? What is the relation between the sides of these triangles?
- An interesting activity to do with the geoboard is to discover the Pick's theorem. The steps for this activity are described briefly:
  – Make different polygonal shapes on the geoboard. Can you determine the areas of these polygonal figures?
  There is a simple formula for obtaining the area of a polygonal figure on the geoboard. It is not difficult to derive this formula. Try to!
  – First begin with a simple figure that is easy — the unit square. What happens to the square as you deform one of its four sides, keeping its other three sides intact? Make a change in one of the sides of the square and describe the result by answering the questions below:
    * What is happening to the number of boundary points? Is it increasing or decreasing or remaining the same?
    * What is happening to the number of points in the interior the polygon? Is it increasing? Are any of the boundary points becoming interior points?
    * Prepare a table for each step as you change one side of the square. Similarly start with a rectangle and observe what happens. Can you arrive at a general rule for the area of a polygon?

The general rule for areas of polygonal figures is given by the formula
$A = \frac{B}{2} + I - 1$. A is the area of the figure, B the number of boundary points, and I the number of interior points.

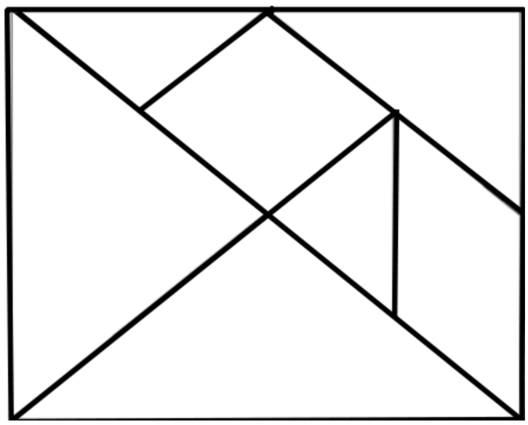
Tangram

Materials required: Card paper.

A Tangram set has seven pieces which are cut from a square piece. The shape and pattern of these pieces is shown in the figure. Tangram puzzles were first developed by Chinese over a thousand years ago. Tangrams are excellent for children to work with in developing concepts of geometric relations.

To make the tangram, cut a square piece of card paper (about 10”× 10”). Cut it in smaller pieces as shown in figure. The tangram pieces can be arranged to form hundreds of different shaped animals and human figures.

TANGRAM

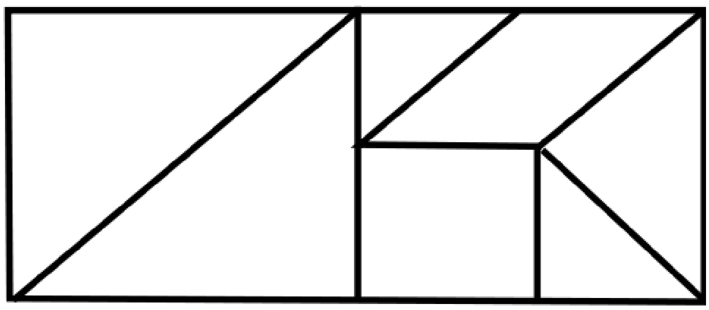


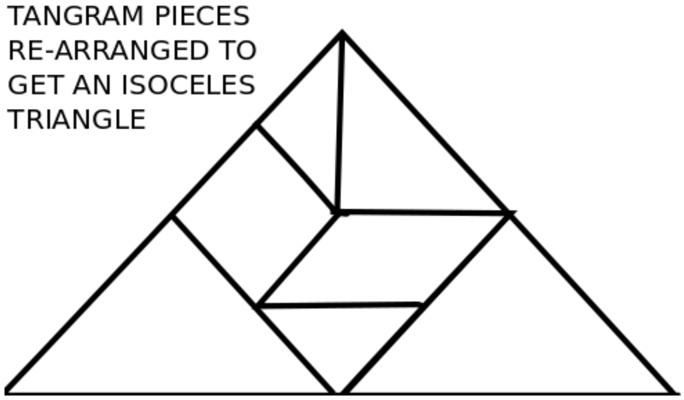
Tangrams can also be used to study geometric shapes. Some activities which can be performed using tangram are the following:

- List the different kinds of figures present in the tangram. How many pairs of equilateral triangles are there in the set?
- What is the area of each tangram piece and of the whole tangram set taking the area of the smallest triangle as one unit?
- Take two small triangles and put them together in such a way that we get a square, a triangle and a parallelogram which are all are equal in area.
- Repeat the above activity with two big triangles.
- Make a square, a triangle and a parallelogram using two small triangles and a medium sized triangle.
- Make a rectangle and a trapezium using two small triangles and a parallelogram.
- Make a rectangle, a trapezium and a parallelogram using two small triangles and a square.
- Make a square, a triangle and a parallelogram using two small triangles, one medium sized triangle and a big triangle.
- Make a square, rectangle and a triangle using all seven pieces.
- Find the area of a given combination of figures taking the area of smallest triangle as one square unit.

- Proof of theorems such as the midpoint theorem, properties of a parallelogram can be discussed using tangram pieces.

TANGRAM PIECES RE-ARRANGED TO
GET A RECTANGLE

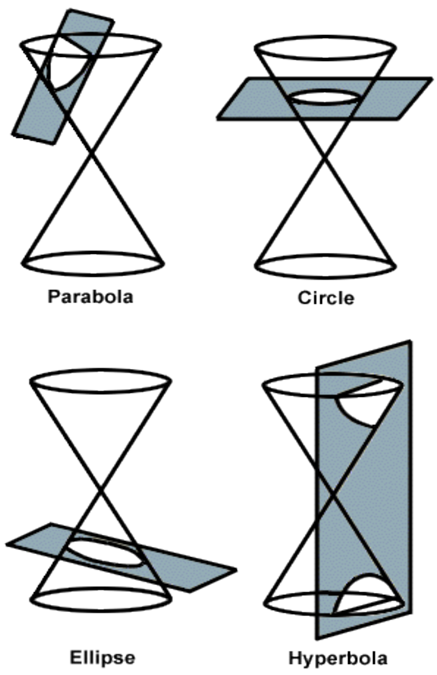




Conic Sections

Materials required: Modeling clay, chart paper, tracing paper, thin wire, divider, plane paper, pen, wooden board, nails string, filter paper, drawing pins, thread.

By cutting through a cone at different angles to the base we can produce a family of interesting geometric curves are called conic sections. These curves are circle, parabola, hyperbola and ellipse.



Conic sections using modeling clay:

If a hollow cone is cut open by cutting along its slant edge and unrolled, we get a sector of a circle. Hence a cone can be rolled from a portion of circular paper. Cut a piece of chart paper into a circle using a divider. Cut away approximately a quarter of a circle by cutting along radial lines, retaining three quarters of it. This can be rolled into the shape of a cone, and the overlapping portion can be pasted or stapled together. Now make an identical cone in the same way from butter paper or tracing paper, but do not staple or paste it into the final shape. Place it inside the chart paper cone to form an inside layer. Now fill the cone with modeling clay. Press it down till it take a shape of a cone. Let the clay cone with the butter paper cover fall out of the paper cone. Peel off the butter paper.
The cone is now ready for cutting.

Stretch a thin wire holding the ends with both hands till it is taut. Use this wire to cut the cone at different angles to the base.

- A cut parallel to the base gives a circle at the section.
- A cut parallel to a sloping side gives a parabola.
- An ellipse is made by cutting through the cone at a slant.
- A cut parallel to the axis of the cone gives a hyperbola.

Drawing an ellipse on board with two pegs or nails

A simple way to produce an ellipse is to use a thread and two nails. First hammer the two nails at certain distance. Now form a loop from the string about three times as long as the distance between the nails and place it across the nails. Now put the point of the pencil inside the loop and move it around the pins, keeping the pencil slanted so that the line is as smooth as possible. The curve that is obtained is an ellipse. Here the two nails form the foci of the ellipse. Ask the students why the sum of the distances from the two foci of every point on the ellipse is constant.

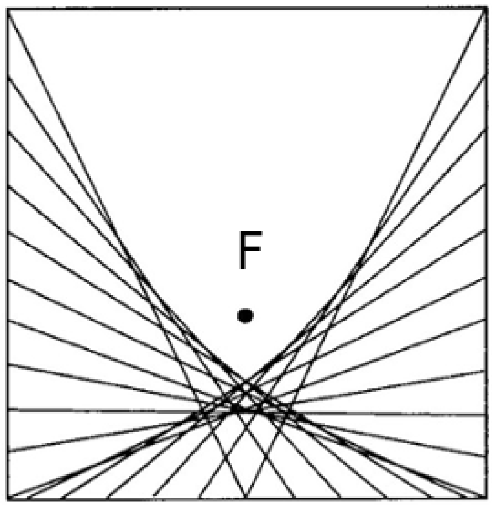
Conic sections through paper folding

One can obtain a beautiful ellipse by folding a circular piece of paper like the filter paper. Mark any point in the interior of the filter paper (preferably about 1 cm from the edge. Now fold the paper so that the circumference falls on F and press it down to crease it. Let the point on circumference which coincides with F be P_1. Take the adjacent point on the circumference P_2 (say about 1 cm away from P_1) and fold the paper again so that P_2 falls on F. Crease the paper once again. Similarly fold the paper repeatedly so that a large number of points on the circumference fall on F. The whole circumference must be covered in this way. Examine the pattern of creases obtained A beautiful ellipse appears in the middle as an envelope of all the creases. The point F and the center of the circle form the two foci of the ellipse.

A parabola can be obtain from a rectangular piece of paper in a similar manner. Mark a point F near one of the edges of the paper. Now fold repeatedly so that the edge close to F falls on F. The pattern

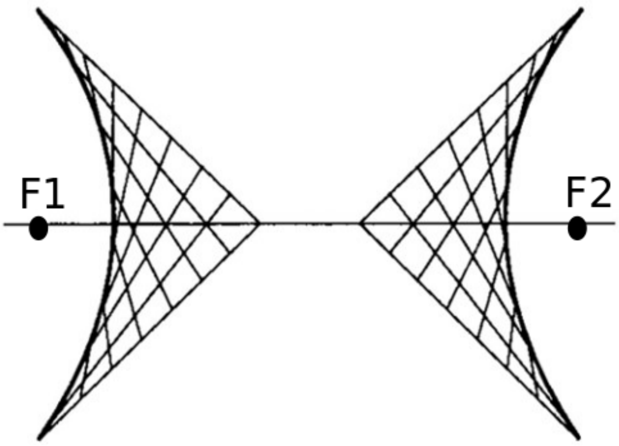
ELLIPSE BY PAPER FOLDING

of creases that emerge show a parabola. The point F forms the focus of the parabola and the edge which falls on F while folding forms the directrix of the parabola.



PARABOLA BY PAPER FOLDING

To fold a hyperbola take a piece of tracing paper (or some transparent paper which is not too thin and which creases well). Draw a circle on the paper and mark a point F outside the circle. Fold the paper repeatedly so that F falls on different points on the circumference of the circle. A pattern of two hyperbolas is obtained each of which is a reflection of the other. The center of the circle and the point F form the two foci of the hyperbola.



HYPERBOLA BY PAPER FOLDING

Making a hyperbola with drawing pins and thread

We can make a hyperbola with thread and drawing pins. In this activity we push about six drawing pins into a piece of cardboard so that they lie at equal distances on a straight line. Make another identical line of drawing pins with the same gaps as the first line. The two lines must intersect to make an angle. Now with a piece of thread join the first pin of one line to the last (sixth) pin of the other to form a line segment between the pins. Join the second pin of the first line to the fifth pin of the second line. Continue joining the pins in this manner to obtain the hyperbola shown in the figure.

Exercise

In the paper folding activity for constructing conic sections, proof of the following can be given as exercise to the students:

1. The sum of the distances of any point on the ellipse from the two foci is constant. **Hints:** Each of the folded creases is a tangent to the ellipse. Study what happens with a single tangent. Folding creates a reflection with respect to the folded line of point F on the circumference. An important step is to identify the point on the tangent which touches the ellipse.
2. The sum of the distances of any point on the parabola from the focus and the directrix is constant.
3. The difference of the distances of any point on the hyperbola from the two foci is constant.

Reference:

Scher, D.P.,"Folded Paper, Dynamic Geometry and Proofs: A Three-Tier Approaches to the Conics", *Mathematics Teacher*, (89, March 1996):188-193.

Surface area and volume

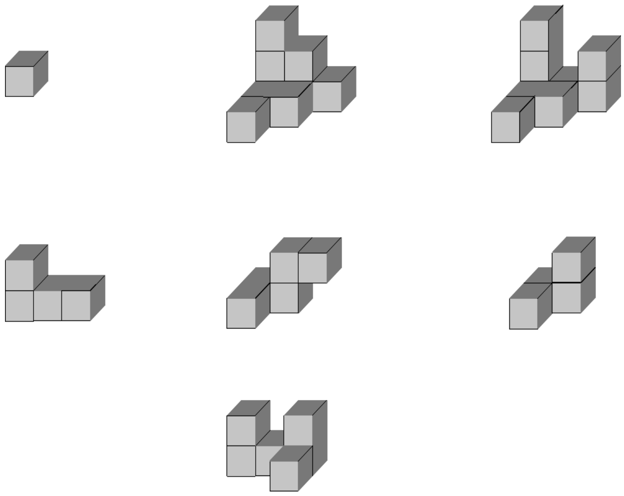
Materials required: eight or more wooden or plastic cubes or alternatively chart paper from which cubes can be made.

In the school curriculum students are introduced to solid geometry through the topic of mensuration. They need to compute the surface area and volume of a number of regular solids. Often students have not fully grasped these concepts of surface area and volume or appreciated the difference between the two. In this activity students compare the surface area and volume of shapes formed by piling identical cubes in different ways. The activity can also extended to explore spatial relationships using the cubes.

For this activity we need eight or more identical cubes of side 1 cm (or 1 inch or of suitable side taken to be 1 unit). The volume of each cube is then one cubic cm. (or one cubic unit). The surface area of each cube is one square cm. (or one square unit) and the total surface area as $6 \times 1 = 6$ sq.cm. (or 6 sq units). If wooden or plastic cubes not available, the cubes can be made from chart paper or from a thin block of soft wood. If chart paper is used, one can make cubes of five sides, with one side open which are easy to make. Five sided-cubes are sufficient for the activity.

The following activities may be done with the cubes:

CUBES WITH DIFFERENT SPACIAL ORIENTATIONS



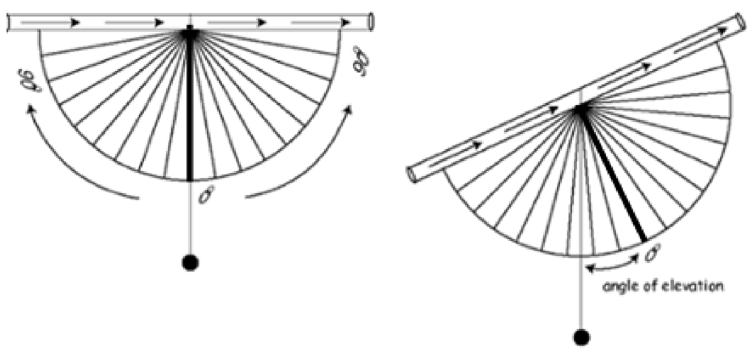
- Ask the students to first find out the total volume and area of all the right cubes.
- Give the students five cubes and ask them to arrange in all possible ways. The cubes must be arranged so that when two cubes make contact, the faces in contact must completely cover each other. For each arrangement let them find the surface area and volume.
- Repeat this activity with 6, 7 and 8 cubes. Find each time the surface area and volume.
- While making each of the arrangements with the cubes the students can be asked to project the plan and the elevations from different sides of the solid figure.
- Given the two dimensional drawings of the plan and elevation, other students can be asked to build up the arrangements using the cubes.

Astrolabe

Materials required: Chart paper, pen, divider, piece of string, a small heavy object to be tied as a bob to the string, hollow tube made of paper or curtain rod about 10 inches long.

An astrolabe is an instrument used commonly in ancient and medieval astronomy to find the angular distances of stars and planets. It can also be used to find the angle of elevation of mountains, trees, buildings or other tall objects.
You can make an astrolabe from a chart paper cut into a semi- circle of about 5 inch radius. A divider or a pair of scissors can be used to cut the chart paper in the form of a semi-circle. Mark the angles from 0^0 to 180^0 on the semi-circle similar to the markings on a protractor. Roll a rectangular piece of chart paper into a hollow tube and stick it with cello tape to prevent it from unrolling. Attach the paper-tube, or if it is available a hollow curtain rod, along the diameter of the semi-circle. From the centre of the circle (which is also the centre of the hollow tube) suspend a plumb-line (bob tied to the string) as in the figure.

ASTROLABE



To find the angle of elevation of a tall object, look through the tube at the top of the object. The angle made by the plumbline with the 90^0 line on the semi-circle is the angle of elevation of the object. If this angle is, then the height of the object above eye-level can be obtained by multiplying the distance of the object by tanθ .

Reference:

Vorderman, C. *How Mathematics Works*, Kindersley, D., London (1996):97

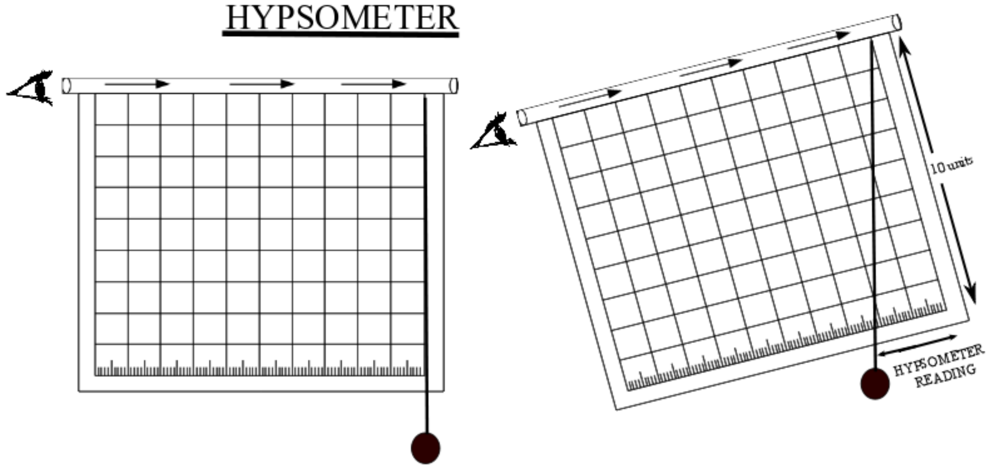
Hypsometer

Materials required: cardboard, graph paper(optional), hollow chart paper tube or curtain rod about 10 inches long, string and small heavy object to be tied to the string as a bob.

This is a modification of the astrolabe, which uses the property of similar triangles to estimate heights of tall objects. With this change it becomes much easier to estimate the heights since we do not use trigonometric ratios. So it can also be used by a student who is not yet introduced to trigonometry.

To make the Hypsometer first cut the cardboard into a rectangle of size 10” x 11”. Paste a sheet of graph paper in inches or plain paper on the cardboard. Draw a 10” x 10” grid on the cardboard as shown in the figure. Graduations in tenths of an inch are marked on the bottom line of the grid. (This is not necessary if one uses a graph paper with inch squares.) Now fix the hollow tube at the top on the first row and suspend the string with the bob from the tube at the first point, that is at the right hand top corner of the grid, as in the figure.

HYPSOMETER



To use the hypsometer, hold it parallel to the ground such that the string covers the first line (which therefore becomes vertical). Now looking through the tube point it to the top of the object whose height is to be estimated. Ask your friend to note the reading where the string crosses the bottom line of the grid drawn on the cardboard. This is the distance from the right most vertical line and is the Hypsometer reading. Now using the following formula we can find the height of the object from your eye line:

height = $height = \frac{hypsometer\ reading}{10} \times distance\ of\ object$

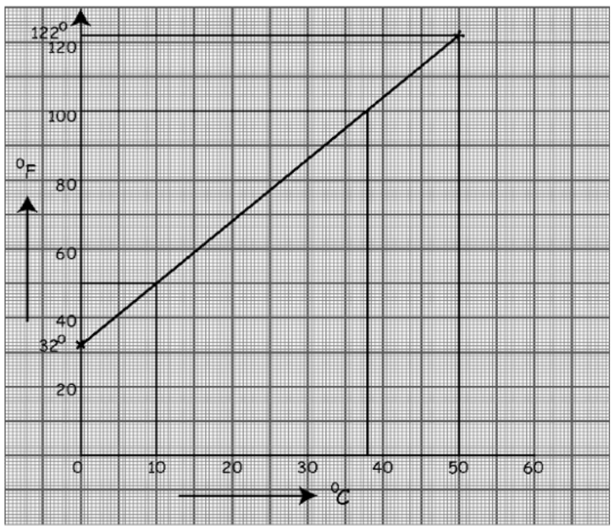
Add your height to this, which gives the height of the object from the ground.

Conversion chart

Materials required: Graph sheet and pen.

Students in the high school often need to convert temperature measurements from the Celsius scale to the Fahrenheit scale and vice versa. This activity aims at making a chart for converting from one scale to another.

CONVERSION CHART



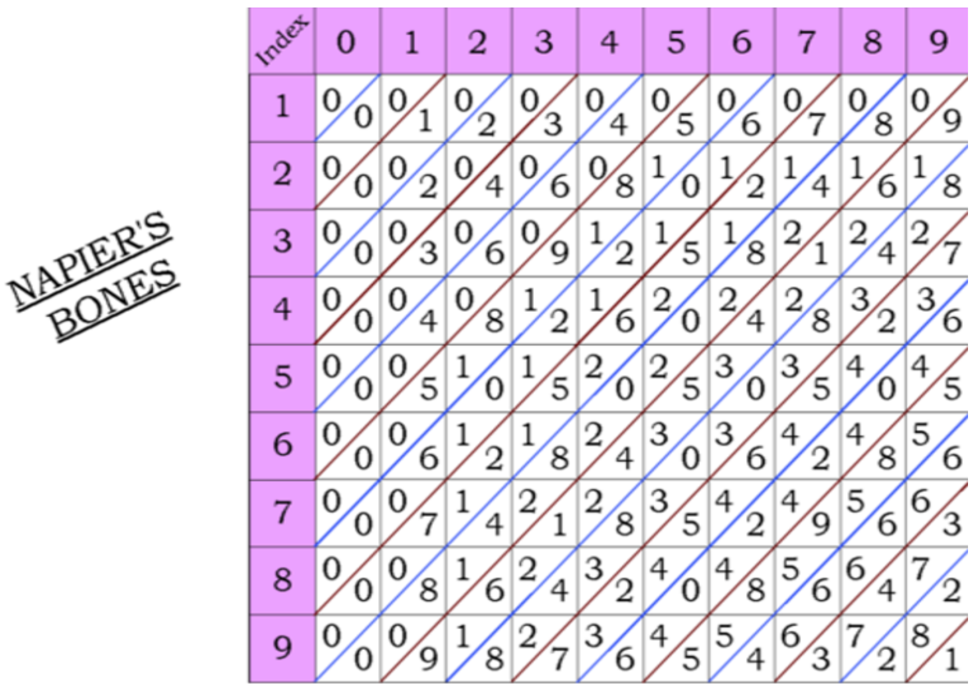
Ask the students to plot the different scales on the X and Y axis on a graph paper. Let them obtain the conversion for three different values of temperature using the formula $C=\frac{5}{9}(F-32)$. Ask them whether the three points lie on a straight line. Let them draw a straight line through these points. Are the temperatures of the ice point and steam point shown correctly in both degree Celsius and degree Fahrenheit by the straight line? Ask the students to verify whether the line can be used as a chart to convert any measure of temperature from Celsius to Fahrenheit. Why is this possible? The chart provides an occasion to discuss the property of linear functions. Let the students explore making charts of other linear functions such as converting inches to cms, or a chart for finding the father's age from the son's age and so on.

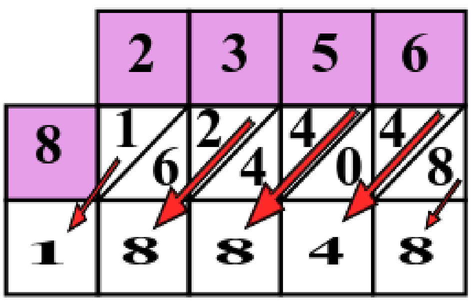
Reference:
Vorderman, C. *How Mathematics Works*, Kindersley, D., London (1996):37

Napier's bones

Materials required: Cardboard or chart paper.

John Napier (1550-1617), the man who invented the logarithmic tables also made this simple calculation device. Napier's bones can be used for multiplying a large number with a single digit number. In this way it reduces multiplication to a series of additions.





Take nine long strips of chart paper or cardboard, 1 inch wide and 9 inches long. Copy the numbers shown in the figure on the nine strips in the manner shown along with the diagonal lines. Now, to multiply 2356 by 8, take the bones with the digits 2, 3, 5 and 6 and lay them next to each other. On the 8th horizontal row add the digits that are formed along each diagonal to get each digit of the answer.

The method of adding along the diagonals used here is very similar to the **Gelosia method** of multiplication developed by Indian mathematicians many centuries ago. It is also quite similar to the method of multiplication taught at school. On observing closely one finds that each of Napier's bones is simply the multiplication table for that number (verify this from the figure). The digits in the units place and the tens place are written separately. So while using Napier's bones, one need not make the effort of recalling multiplication tables. Addition of digits, carrying over the digit in the tens place are managed by the diagonal arrangement.

Reference:
Voderman, C., *How Mathematics Works*, Kindersley, D., London(1996):23

Nested Platonic solids

Materials required: Drinking straws preferably the harder variety, rounded paper clips, needle and thread.

A platonic solid is a solid whose faces are regular polygons. All its faces are congruent, that is all its faces have the same shape and size. Also all its edges have the same length. Platonic solids are regular tetrahedra.

The most common platonic solid is the **cube**. It has six faces and each face is a square.
Another platonic solid is the regular **tetrahedron** which has four faces and each face is an equilateral triangle.

How many such Platonic solids are possible? It is an interesting fact of geometry that there are only five possible platonic solids. Apart from the cube and the tetrahedron, there is the **octahedron**, which has eight faces, all equilateral triangles, the **dodecahedron**, which has 12 faces, all pentagons and the **icosahedron** which has 20 faces, all equilateral triangles. This was known to the Greek philosopher Plato, who lived over 2000 years ago, and after whom the solids are named. Plato thought that the Platonic solids held the key to the structure of the heavens, something which the astronomer and physicist Kepler, who lived in the 17th century also believed for a while.

To make the Platonic solids, take straws of equal lengths and join them in the shape of the Platonic solid. The joints are made from rounded paper clips which can be bent to any desired angle at the junction. The two 'U' shaped arms of the clip are pushed into the opening of straws. Open up these arms a little for a better fit.

All these Platonic solids, if hollow, can be fitted exactly or nested one within the other. Some of the nesting combinations are interesting since the corners of the inner Platonic solid touch the vertices or the edges of the outer solid. One such sequence of combinations, with all the five platonic solids nested one within the other is interesting.

In this sequence the outer most platonic solid is the Dodecahedron. A cube can be nested within the dodecahedron. The twelve faces of the Dodecahedron are regular pentagons. On one of the faces join two opposite corners. This forms one edge of a **cube** which can be nested inside dodecahedron. The edge-length of the dodecahedron is 0.618 times the edge-length of the cube. These two nested solids can be made again with straws and paper clips.

By joining one diagonal on each face of the cube, one obtains a tetrahedron. The length of the edge of the tetrahedron is then √2 times the length of the edge of the cube. This nested combination can also be got from straws and paper clips.

Joining the mid-points of all the edges of the tetrahedron gives an octahedron. The edge of the octahedron is then half the edge of the tetrahedron in length. To make this combination, first make the tetrahedron and mark the midpoints on all the straws. Take pieces of straw which are half the length of the straws on the tetrahedron (a little less than half actually). Now pass a needle and thread through the midpoint of a side of the tetrahedron, slip in the smaller piece through the thread and pass the needle and thread through the midpoint of the adjacent edge of the tetrahedron. Making the thread taut, we find that one edge of the octahedron is in place. Similarly join all the edges of the octahedron with needle and thread.

Inside the octahedron the remaining platonic solid, the icosahedron can be fitted. Divide each edge of the octahedron in the golden ratio 1:1.618 and mark the points. Join the points on adjacent sides in cyclic fashion and you get an Icosahedron. To make this combination with straws, first make the octahedron with straws and paper clip joints. Now take pieces of straw which are ** times the length of the straw forming one edge of the octahedron. (Actually the pieces need to be somewhat smaller so that they fit in nicely.) Mark the point on all the edges of the octahedron which divide the edges in the ratio 1:1.618. On adjacent sides these points must not be equidistant from the vertex, but must alternate in the ratio 1 and 1.618. Pass a needle and thread through these points, thread in the smaller pieces and make the icosahedron. This will require some patience, but the resulting figure will be worth the effort.

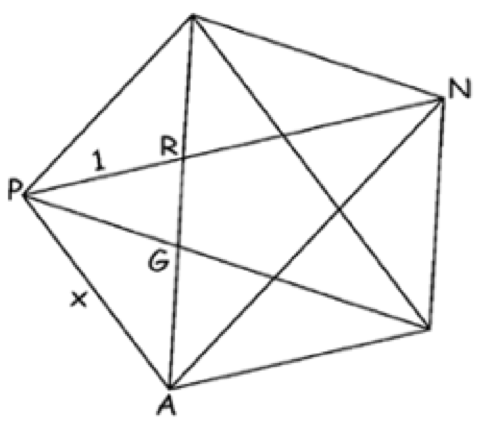
Reference:
Hopley, R.P. "Nested Platonic Solids: A class Project in Solid Geometry", *Mathematics Teacher* (87,1994):312-318

Pentagrams and the Golden ratio

Materials required: thin strip of paper about 2 inches wide, plain paper and pen.

A pentagram is a regular pentagon with all possible lines joining corners drawn inside it, creating a starlike effect. The pentagram has such beautiful symmetry that it has been a powerful symbol in many cultures throughout history.

PENTAGRAM



A pentagram is shown in the figure alongside. Ask students to try and answer the following questions by studying the figure.

1.Consider ΔAPR in the figure. Which other triangles is it similar to?

2. How many triangles can you spot which are congruent to ΔAPR

3. Can you find all triangles congruent to those mentioned in item 1?

4. Can you find the value of x? (Hint: use the similarity relation.) Express x both in the radical form and in the decimal form.

5. Compute the value of 1/x and x^2. What do you notice? Can you arrive at this relation from the expression for x in the radical form?

6. Write a few terms of the Fibonacci series. The series starts with 1. The second term of the series is also 1. To get subsequent terms we add the preceding two terms. The third term hence is 2 and the fourth term is 3. Now, assume that the ratio of successive terms in the series converges to a certain value as the number of terms become large. Can you find this ratio? Use the fact that each term is a sum of the preceding two terms.

It is not possible to construct the regular pentagon with ruler and compass. However there is a simple way in which a regular pentagon and hence the pentagram can be obtained. Take a 2 inch wide strip of paper which is about a foot long. Tie it into a simple knot taking care to see that the paper strip retains its full width inside the knot. Keeping it flat in this way and tying the knot carefully somewhat like a knot in a necktie, one obtains a pentagon. Draw all the diagonals of the pentagon. The resulting figure is a pentagram.

The golden ratio appears many times inside the pentagram. In the pentagram shown here, it is the ratio x : 1. x and 1 are the lengths of two unequal sides of the isosceles ΔPAR.

If you observe carefully, you will find that there are ten triangles in the pentagram which are the same size and shape as ΔPAR. ΔPGR is smaller, but it is similar to ΔPAR. There are five of these small triangles. Finally there is the big ΔPAN, which is also similar to ΔPAR. There are five of these big triangles.

In all these isosceles triangles the lengths of unequal sides bear the golden ratio. From this property the value of the golden ratio can be derived. An approximate value for the golden ratio is 1.618.

One of the peculiar properties of the golden ratio is that its reciprocal is equal to one minus golden ratio $(\frac{1}{x}=1-x)$. Another peculiar property is that the square of the golden ratio is equal to one plus the golden ratio.

A golden rectangle is a rectangle which has its sides in the golden ratio. When three of these rectangles intersect each other at right angles and their mid-points coincide, you can join all the corners and get a regular **Icosahedron**.

The ratio between two successive numbers in the Fibonacci series converges to the value of the golden ratio.

The golden ratio has many connections with art, architecture, natural phenomena and regular decagons.

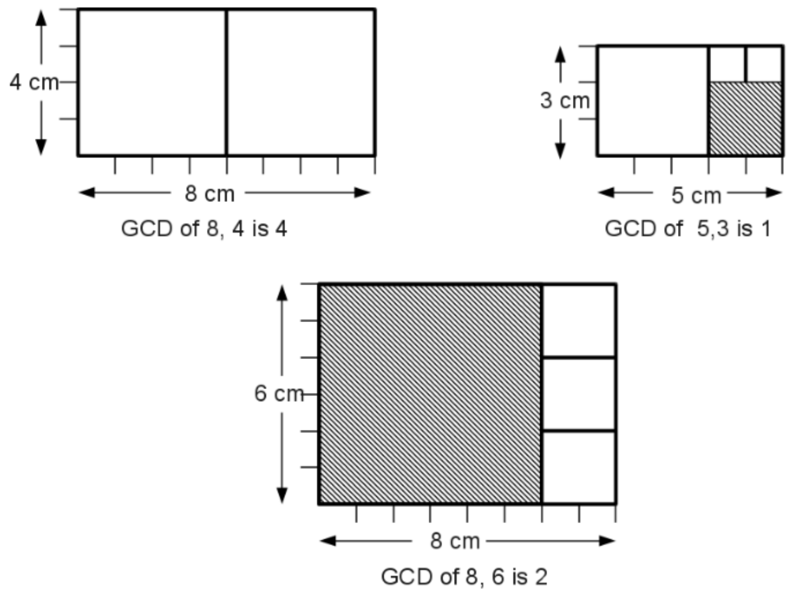
Reference:

Miller, W.A.. and Clason, R.G.,"Golden Triangles, Pentagons, and Pentagrams", *Mathematics Teacher*,(March1994):338-341.

Greatest Common Divisor by geometry

Materials required: a piece of paper, a scale and a pencil.

The G.C.D. of a pair of integers can be found by an interesting geometric method. (G.C.D. of a set of integers is the largest integer which can divide all the numbers without remainder). In order to do this, we only need a piece of paper, a scale and a pencil.



Suppose the two integers whose G.C.D. is to be found are a and b. First draw a rectangle on the paper of length a and breadth b. (If a and b are large take the length and breadth as $\frac{a}{2}$ and $\frac{b}{2}$, or in some suitable proportion).
From this rectangle mark off the largest possible square. If a is greater than b, this will be a square of side b. After marking off the square, the portion which remains is a rectangle with sides b and $a - b$. Again mark off the largest possible square from this rectangle. Continue this process till you obtain a square instead of a rectangle. The measure of the side of this square is equal to the G.C.D. of the original pair of numbers.

This method is based on the fact that if a and b are both divisible by a number, then $a - b$ will also be divisible by the same number. The same principle is applied recursively to obtain the G.C.D.

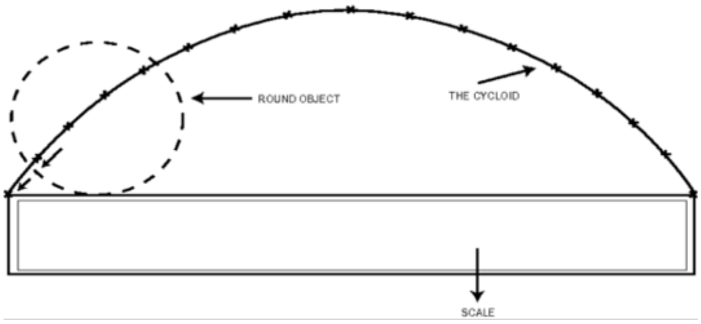
Special geometric curves

Materials required: Ruler, carom striker, pen, mild steel wires for double helix.

Many types of interesting geometric curves occur in nature. It is possible to draw these curves using simple tools. We have included here techniques for drawing a few of these curves.

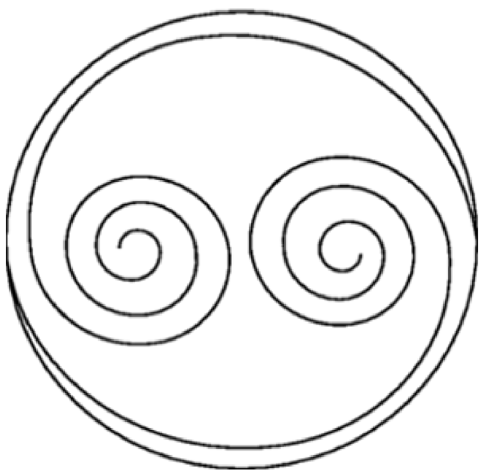
Cycloid: To draw this curve, place a ruler on a sheet of paper and fix with cello tape. Draw the edge of the ruler on the paper. On a carom striker mark a point on the circumference with a pen. Place the striker against the ruler on the paper, so that the point marked touches the ruler. Now without sliding the striker, carefully roll it on the edge of the scale till it moves forward by 2 cm. Now mark the point on the paper just below the point marked on the striker. Roll the striker forward again carefully by another 2 cm. Again mark the point on the paper under the point marked on the striker. Repeat this till the striker completes a full cycle and again touches the ruler. Remove the ruler and striker and join all the points marked on the paper by a smooth curve. This curve is a cycloid.

A CYCLOID



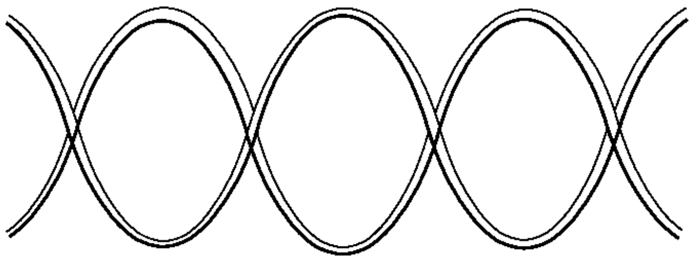
Celtic spiral: Celtic spirals were drawn by an ancient people called the Celts who migrated across Europe to decorate their art. To draw a celtic spiral, draw a circle with a radius of 8cm. With a light pencil draw a diameter of the circle. Mark off one cm parts on the diameter, so that there are 16 parts on it. Now with the free hand try and draw the pattern of spirals shown in the figure. Note that both the spirals are clockwise. One starts from below the diameter and another from above the diameter.

CELTIC SPIRAL



Double helix: A helix is a three dimensional spiral that looks like the thread on a screw. It is actually a line going at a constant angle around a cylinder or a cone. A common spring 25 is a simple example of a helix. A double helix has two similar helices wound round each other. This structure is one of the most important structures in the real world, for it is the structure of the DNA molecule.

DOUBLE HELIX



To make a helix, take a tube of cardboard or metal, or a cylinder which is easily available. Take a rectangular piece of paper which exactly rolls on the cylinder. On the rectangular paper, start from one corner (say point A) and draw a line at any angle such that it cuts two longer sides of the rectangle (the sides corresponding to the height of the cylinder) at points A and B. Now mark a point C exactly opposite to point B on the same side as point A. Draw another line parallel to the first line. Continue this process till the lines go out of the rectangle.

Now roll the paper on the cylinder so that the lines are visible. The lines form a helix on the cylinder. Roll a piece of wire made of mild steel (which is commonly available in hardware shops) on the line and press it down so that it takes the shape of a helix.

To make a double helix make two identical helices with wire. These can be mounted on a wooden board to represent a DNA molecule. Small balls made of modeling clay can be placed on the two helices to represent the DNA bases.

Reference:

Voderman, C., *How Mathematics Works*, Kindersley, D., London(1996):146-147.

Soap Films

Materials required:

Soap films provide fascinating material for mathematical study. It is possible to obtain very interesting and perfectly shaped three dimensional figures using soap solutions which otherwise are difficult to create or to imagine.

To make the soap solution, pour two handfuls of blue detergent powder into half a bucket of water. If the water is hard in your area, then you need to use distilled water. Add about 25-30 ml of glycerin which strengthens the soap films obtained.

Now make wire frames using mild steel wire in different shapes. If two circular loops held one above the other are dipped in the soap solution a film stretches across both of them. By pulling the loops apart, we obtain a three dimensional shape called a catenoid. A cubical frame dipped in soap solution and pulled out carefully reveals a beautiful pattern of surfaces. There is a small cube of soap film present in the centre which is connected to the edges by planes. The figure is a projection of a 4-dimensional hypercube in three dimensions. A frame which is shaped like tetrahedron dipped in the solution similarly shows a small tetrahedron of soap film in the center. However the tetrahedron in the center has beautiful curved surfaces and edges. When bubble is placed between two parallel transparent plates (made of acrylic or glass), the bubble transforms into a perfect cylinder. It is quite interesting to examine what happens when more bubbles are placed between the plates.

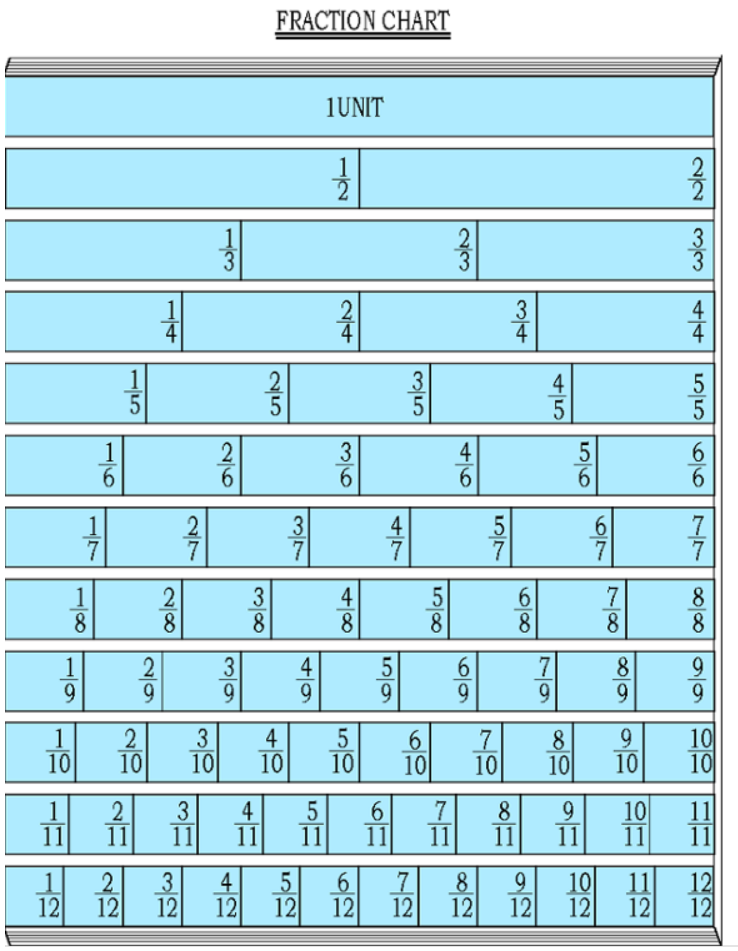
Reference:

Voderman, C., *How Mathematics Works*, Kindersley, D., London(1996):142

Fraction chart (A teaching aid)

Materials required: Plywood or cardboard, Chart paper, string, and bob

The concept of fractions of a whole is introduced in primary school. Students find it difficult to master and often even understand the concept of a fraction and the meaning of the numerator and denominator. A fraction chart is a very useful teaching aid which can be used for this purpose.



A fraction chart is made from a piece of plywood or thick cardboard which is large enough to be put up on the wall. Narrow strips of chart paper of equal length are pasted on the board at equal distances. Let the first strip represent 1. Divide the next strip into two equal halves and mark the fractions $\frac{1}{2}$ and $\frac{2}{2}$. Divide the next strip into three equal parts and mark the fractions $\frac{1}{3}$,

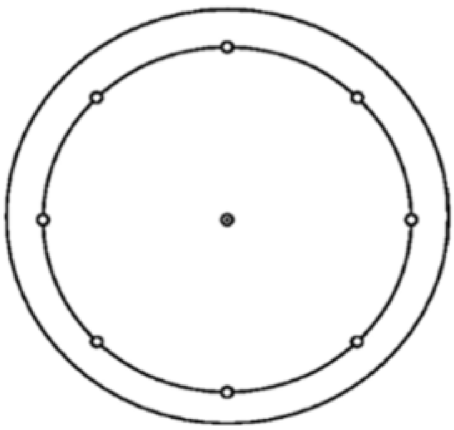
$\frac{2}{3}$ and $\frac{3}{3}$. Continue in this way till all the strips are divided to obtain smaller and smaller fractions. One can make a chart till the fractions $\frac{1}{20}$, $\frac{2}{20}$ . . . if space is available. Now suspend two long strings from the top of the board with bobs attached at the end. The strings remains vertical like a plumbline.

The fraction chart can be used for showing the part whole relationship: how many one thirds make up one? Another important use of the chart is to show equivalent fractions. Drop the plumbline over a fraction, and if the chart is aligned vertically all the fractions which coincide with the plumbline are equivalent fractions. The students also learn that any fraction of the form $\frac{n}{n}$ is equal to 1. It is also possible to do some rudimentary addition and subtraction of fractions with the chart. If two fractions are to be added find their equivalent fractions on the same line of the chart by dropping the plumbline. Now it is possible to add the fractions easily by adding the numerators.

Property of circles (A teaching aid)

Materials required: Plywood or acrylic board cut in the form of a circle, pegs, rubber bands.

This is a teaching aid which can be used to illustrate the angle properties of a circle. These properties are generally not obvious to most students.



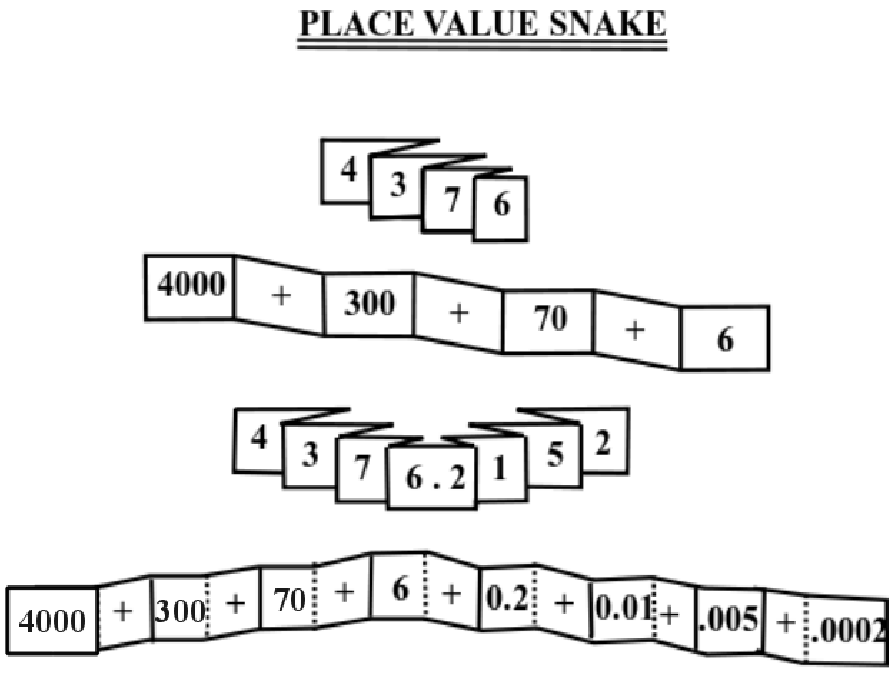
The teaching aid consists of a circular piece of plywood or acrylic of about 8 inch diameter. An inner circle of about 7 inch diameter is marked out on the acrylic or wood. A few holes are drilled on the inner circle and at the centre as shown in the figure. At least some of the holes on the circle must be diametrically opposite to each other.

Put pegs on two holes which are diametrically opposite to each other and stretch a rubber band over the two pegs. With the third peg on any other hole, measure the angle subtended by the diameter on the circumference. By stretching a rubber band over four pegs on the circumference verify the angle property of a cyclic quadrilateral. Similarly verify the equality of angles in the same segment. The alternate segment theorem can also be demonstrated with this teaching aid.

Place value snake

Materials required: Strip of paper, pen

The concept of place value is not easy to grasp for very young children. The place-value snake is a delightful activity which helps in fixing the concept in the mind of a young child.



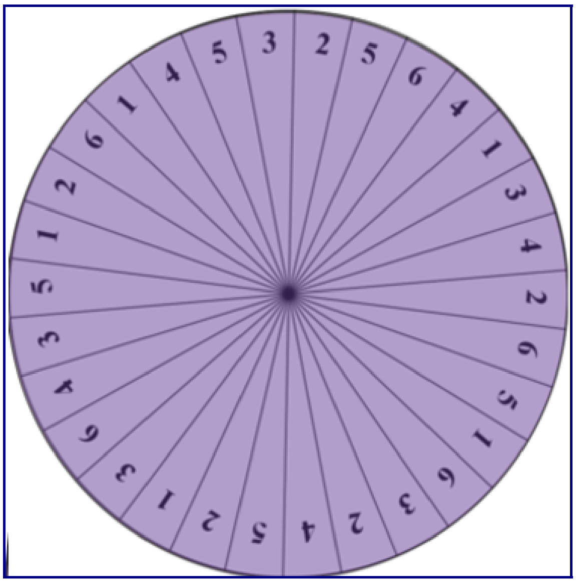
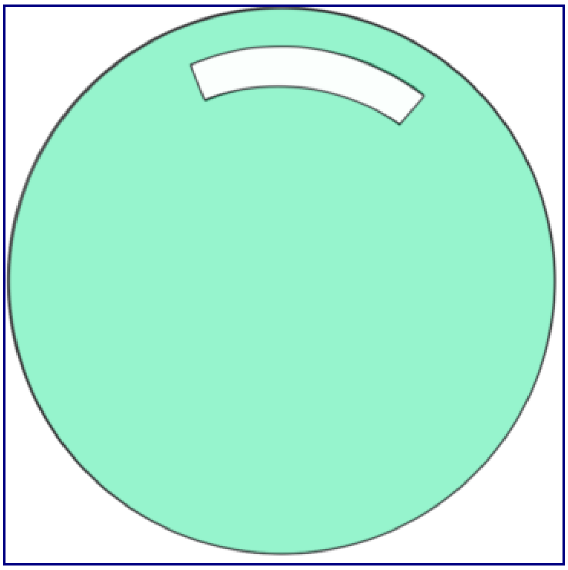
To make a place value snake, take a strip of paper and fold it in the manner shown in the figure. Write a number, say 4376, on the visible part of the folds as seen in the figure. Hidden away in the folds of the paper are zeros and '+'s. When the snake is opened, the expanded form of the number appears.

Any child can make the place value snake and show it to his friends. In the course of doing this he learns to associate the place of a digit with a power of 10. The snake can also include decimal numbers as shown in the figure.

Seven divisibility disc

materials Required: card paper, pen, pencil, scissors

Seven divisibility disc contains two discs. First disc contains a no. of divisions of a circle and each division contains some digits. The second one has a hole. When the second disc is put on the first certain no. of digits(say 5) can be seen.

Now rotate the second disc on the first disc. Number that can be seen through the hole will be divisible by 7. How it happens?

If you want to make a n digit seven divisibility disc, then the number of divisions you need to make on the first disc are equal to the LCM of n and 6.

For example to make a 5 digit seven divisibility disc first you need to make 30 divisions on the first disc and write a four digit arbitrary number in the four consecutive columns. Divide it by 7 and add the fifth digit to make the 5 digit number divisible by 7. Now take the last four digits if the 5 digit number and add fifth digit to that number to make a number which is divisible by 7. Continue the process and it can be found that after thirtieth digit the same numbers reappear in the same sequence.

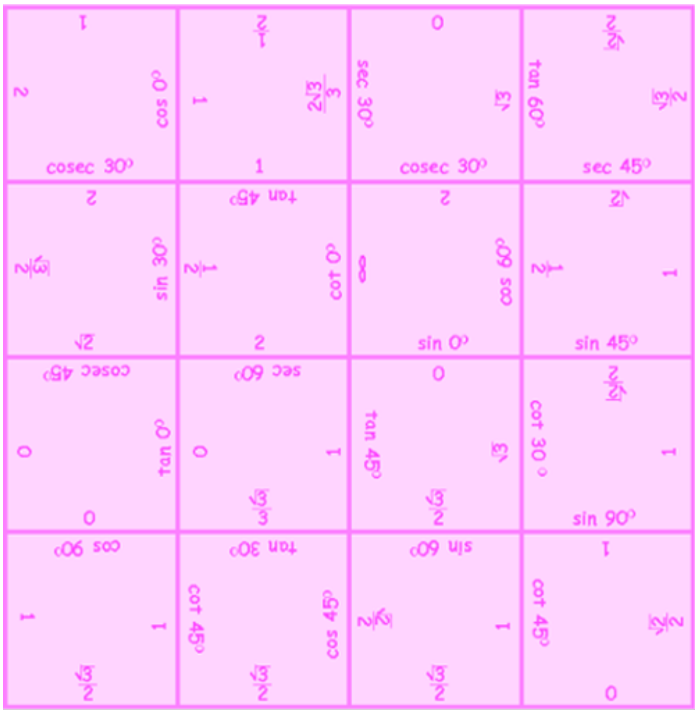
PART II
Puzzles and Games

Trigonometry puzzle

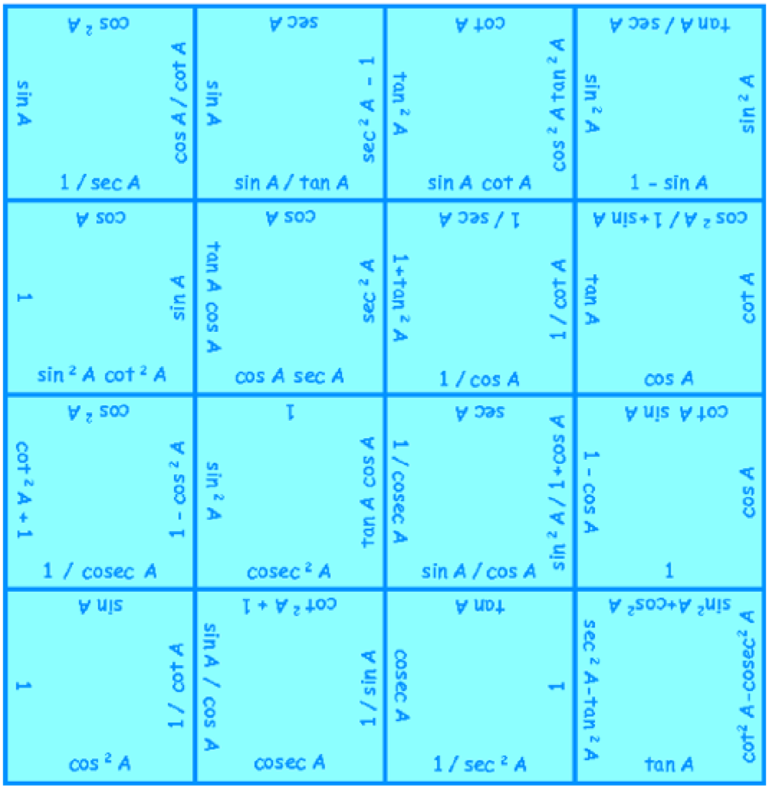
Materials required: Strip of paper, pen

This is a puzzle similar to a jigsaw puzzle, where students have to put together 16 square pieces to form a larger square.

All the pieces have trigonometric ratios or numerical values written along their four edges. The pieces will have to be assembled in such a way that at each joint, the trigonometric ratio and the numerical value correspond. Students are easily attracted to the game and can spend a long time attempting to solve the puzzle. A sample chart of 8in x 8in is shown in the figure, from which the 16 pieces can be cut. The level of difficulty can be reduced by marking the outside edges and the corners differently instead of with numerical values.



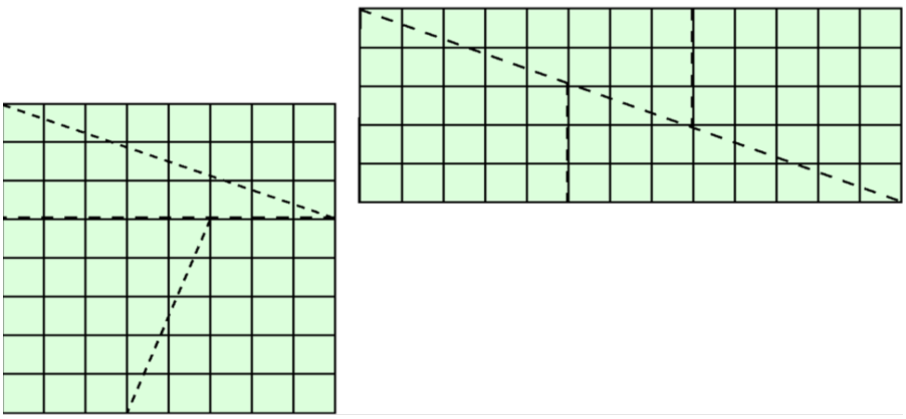
Another chart with trigonometric identities is also shown. After the first game becomes easy, the second chart game can be introduced. It is also possible to design similar charts which can be cut into pieces shaped like triangles or hexagons.

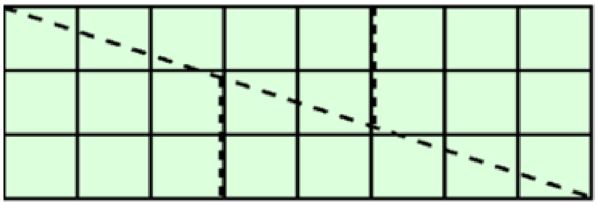
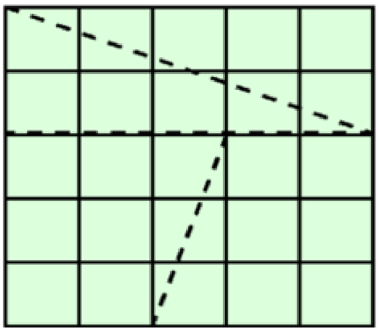


Missing squares and lines

Materials required: Card paper, graph paper(optional), pencil or pen, cutter or scissors.

The missing square and missing line activities take everyone by surprise. But there is a perfectly clear explanation for the mysterious missing square and missing line. The missing square: To make the missing square puzzle, cut a paper into 5” by 5” square. Form 25 smaller squares by drawing lines for every inch. Now cut this square along the dotted line shown in the figure to get four pieces. These four pieces on rearranging into a rectangle give us only 8 x 3 = 24 squares, leaving us surprised as one square is missing from the original piece. The same activity can be performed on a 8” x 8” square and cutting it into four pieces as above. On rearranging the pieces into a rectangle we get 13 x 5 = 65 squares, this time one more than the original.

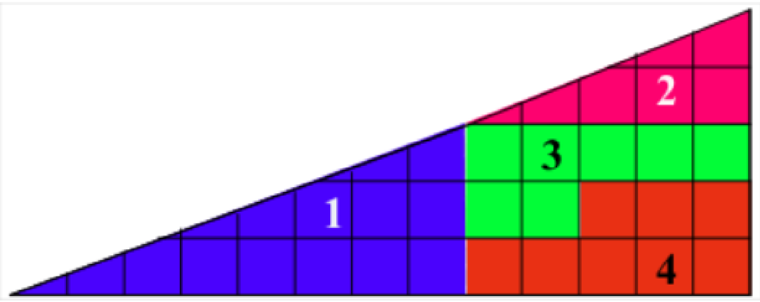


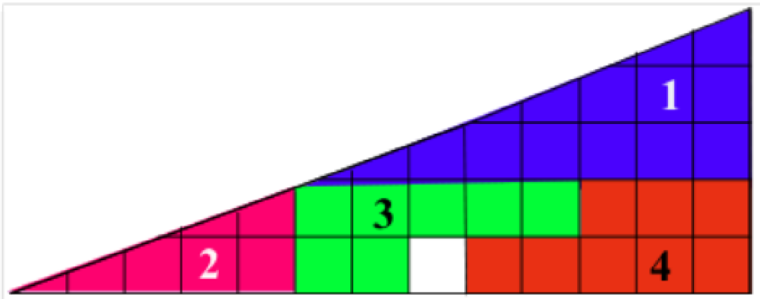
 

The reason for this increase or decrease in the number of square is simple. In each case when we cut the square into pieces and re-assemble them to form the rectangle, the small squares that are obtained are not perfect squares, but are slightly enlarged or compressed. Sometimes the edges do not match. In order to obtain perfect squares, the pieces have to overlap slightly. The total area of overlap will be equal to the area of one square, thus bringing down the number of squares from say 25 to 24. Alternatively gaps may have to be left along the junction lines to get perfect squares. In this case area is added and the total increase in area is equal to the area of one square. The number of squares then increases say from 64 to 65.

The missing square 2: Missing square puzzle contains a right angle triangle with base 13 units and height 5 unit which is formed by four components

1. a right angle triangle with dimensions 8 units x 3 units
2. a right angle triangle with dimension 5 units x 2 units
3. a L shaped figure with 7 sq. Units
4. a L shaped figure with 8 sq. Units





rearranging these 4 pieces to form a new triangle with the same dimension as shown in the figure.

Both the triangles form 13 unit x 5 unit right angle triangle, but the second arrangement has a missing square in it.

Where did the square go?

It can be noticed that the area of both the triangles and the combined area of the components are different.

Area of the components: Area if piece 1 = ½ x 8 x 3 =12 sq.units
Area if pace 2= ½ x 5 x 2 = 5 sq.units
Area if pace 3 = 7 sq.units
Area of piece 4 = 8 sq. Units

Total area of components is 32 sq. Units

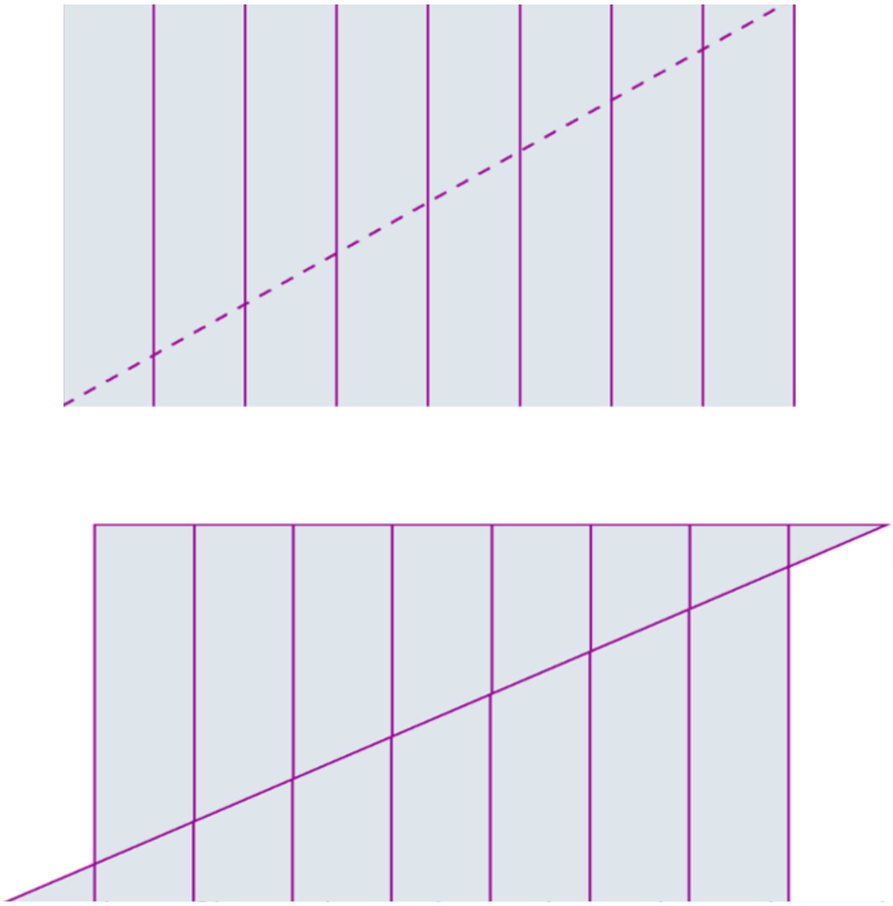
Calculated area = ½ x 13x 5 = 32.5 sq. Units

so calculated area and combined area doesn't match. Why it is so?

Notice the hypotenuse of the piece 1 and piece 2. hypotenuse of piece 1 has a slope of 3/8 where as the hypotenuse of piece 2 has a slope of 2/5. so when combining these two lines don't constitute a single line for both the triangles. The combined hypotenuse in both the triangles are actually bent.

When overlapping both these triangles, overlaying hypotenuses results a very thin parallelogram with area of exactly one square, the same area " missing" from the rearranged figure.

The missing line: If a rectangle as shown in the figure is cut along a dotted line and shifted as shown, one of the original eight line segments inside the rectangle disappears. What happened to the missing line? Tell the students to observe what is happening while sliding the two halves. When we slide the two pieces, one against the other, one line segment in each half becomes the boundary of the rectangle. And the lengths of the remaining line segments are slightly increased, the total length of the eight segments remains the same.



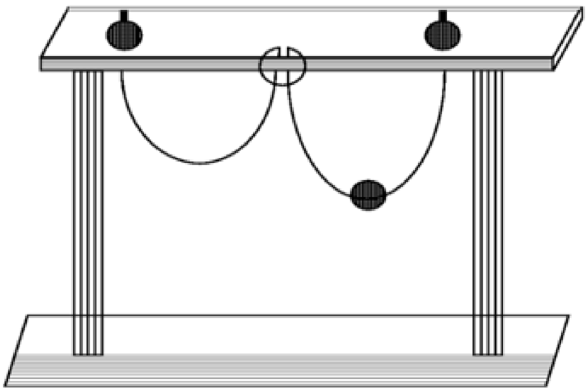
Topology games and puzzles

In this section we have included games based on elementary principles of topology. Most of these games also contain an element of surprise and are entertaining.

BUTTON HOLE PUZZLE*:*

Materials required: a shirt or a piece of cloth with a button hole, some string, a pencil and cello tape.

First thread the string through the button hole as shown in the figure. Attach the pencil to the ends of the string with the tape. The puzzle is now to remove the pencil from the button hole without cutting the string. The solution is fairly simple: the pencil needs to be pulled through the loop in the string and through the button hole.



BELT AND BUCKLE PUZZLE*:*

Materials required: belt with a buckle.

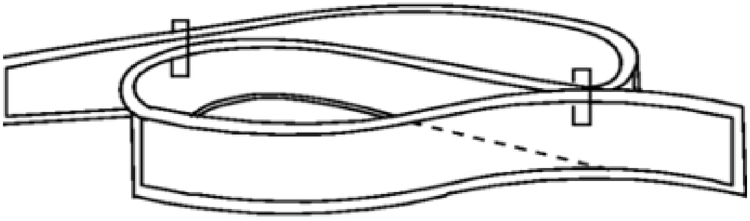
Like the previous puzzle, this is also fairly simple, but may still be new to most students. Many students are familiar with a peculiar change which sometimes takes place in the buckle of an ordinary belt. The pin suddenly is on the wrong side. Children often have no clue about how this happened, or how to get the pin on the right side again, without bending it. This can be posed as a puzzle. The solution lies in taking the belt completely through the buckle once.

LOCKING PAPER CLIPS:

Materials required: strip of paper or transparent plastic, two paper clips.

Take a thin strip of paper about one inch wide, preferably cut from OHP transparency film. Fold it at one end to make a loop and secure it with a paper clip. Fold the other end to make a second loop and secure it with another paper clip. The two loops must make a 'figure of 8' as shown in the figure. Now hold the two ends of the paper strip and pull them apart sharply so that the paper strip straightens out. Watch what happens to the paper clips.

LOCKING PAPER CLIPS

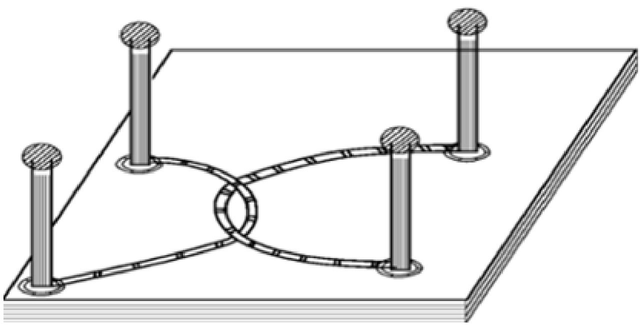


INTERLOCKED LOOPS :

Materials required: Two pieces of string each about 3 feet long.

Tie the first of the two strings to both the wrists of a student.

Let the loop which passes over the wrists be loose. Tie the second string in a similar manner to the wrists of another student, but first allow it to pass under the first string, so that the two strings are interlocked as shown in the figure. The puzzle is to disentangle the two strings without cutting them.

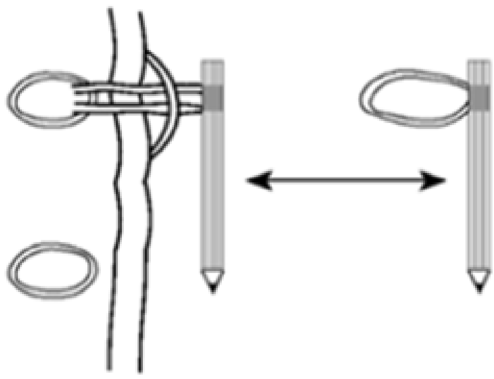


This puzzle is based on the fact that the two strings which look interlocked are not really so. The strings are open at the wrist although the loop appears to be closed here. Imagine that the hand is cut just below where the first string is tied to the wrist. Then it will be easy to take out the second string. Now imagine that the cut moves further and further up the hand. At one point it crosses the loop. Now let the second string which must pass through a cut in the hand also cross the loop. For this you must let it pass through the loop. Now take the second string over the fingers (where the hand is actually cut, that is, where the hand ends) and the strings disentangle easily.

STRING AND BEAD:

Materials required: A long piece of string, three beads and a rectangular piece of wood.

Make three holes of about 1 centimeter diameter in the piece of wood, one in the centre and one near each edge. The beads must be big enough not to pass through the hole. Tie a bead at one end of the string and pass the other end of the string through a edge-hole in the piece of wood. Now take the string through the centre hole so that it forms a loop as shown in the figure. Thread the second bead through the string, take the string out through the other edge-hole in the piece of wood, and tie the last bead to the end of the string. The final arrangement is as shown in the figure. The puzzle is now to take the middle bead to the other loop in the string.



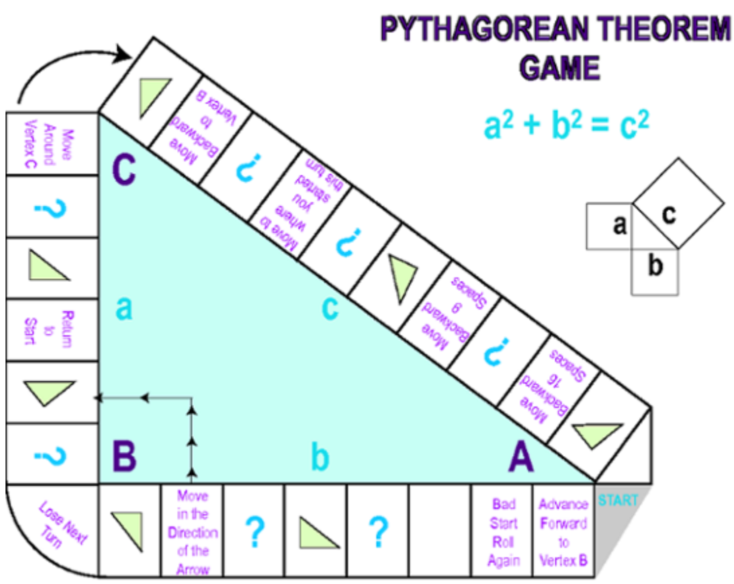
The principle is the same as in the previous puzzle. Hold the loop in the centre of the piece of wood with the thumb and fore- finger, pull it towards one of the edge-holes. Pass it through the edge-hole from below, take it over the edge bead and pull it back through the edge-hole. It becomes partly disentangled. Repeat this at the other edge-hole, and the string comes fully out of the centre hole, forming one large loop. Now move the middle bead to the place where you desire it to be and do the steps in inverse fashion. At the end of the two inverse steps, you are back to the original arrangement, but with the bead now in the other loop.

Reference:
Voderman, C., *How Mathematics Works*, Kindersley, D., London(1996):166-167

Pythagoras theorem game

Materials required: Chartpaper for making the board and question cards, cutter, buttons, a pair of dice

This game is meant to familiarize students with the Pythagoras theorem which plays a crucial role in school geometry.



In this game two or more students roll a pair of dice to move around the triangular board shown in the figure. Copy out the board on to chart paper. Make question cards by copying each question on to a separate card.
Both the dice have to be rolled together. Add the squares of the numbers on both the dice and take the square root (from the famous theorem $a^2 + b^2 = c^2$). Round off the answer to the nearest whole number and move that many spaces.

For example, if you have rolled 1 and 2 on the dice, then $c^2 = 1^2 + 2^2 = 5$. Since $\sqrt{5} = 2.236$ approx., you move 2 spaces.
Follow the instructions given on the square you come to. If your square is marked with a '?', you have to pick a card at random and answer the question on it. If you answer correctly you get a chance to roll a single dice and move the number of spaces it shows. A wrong answer means you stay on the square till your next turn.
The first player who goes twice around the board wins.

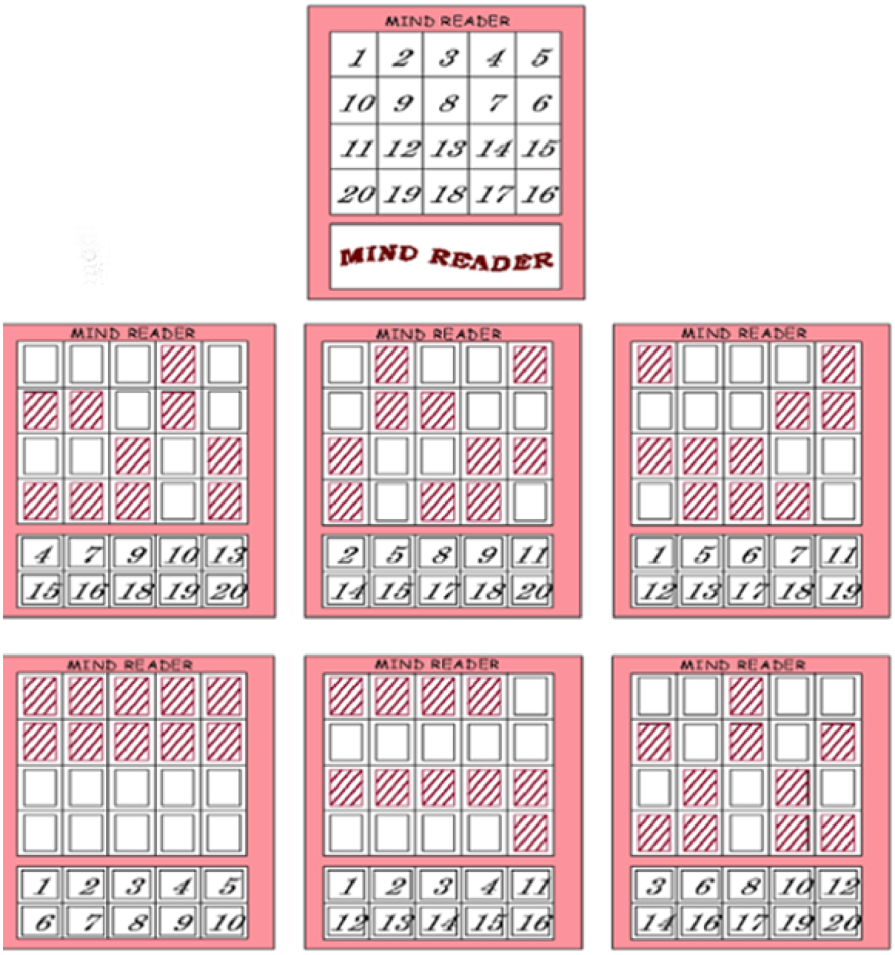
Reference:
Erickson, D., Stasiuk,J. and Frank,M., "Bringing Pythagoras to life", Mathematics Teacher,(Dec 1995): 744-747.

Magic mind reader – 1

Materials required: Chartpaper, pen, cutter.

Here is a simple game to boggle your mind. Cut out seven cards from chart paper in equal sized rectangles (about 8" x 6"). On the main card copy out the numbers 1 to 20 as shown in the figure. On the remaining 6 cards, cut windows in the pattern shown in the figure and also copy down the numbers on the bottom two rows of each card. Now the game is ready.

MAGIC MIND READER



Ask your friend to think of a number from the main card. Let him tell you on which of the other six cards (mask cards) the number appears. There will be three cards on which it appears. Stack these three cards together and place it on the main card aligning the edges. The number your friend thought of appears in the magic window!

How does it work?

There are six mask cards. Any three of them contain a given number. The different ways in which you can select three cards out of six is given by the formula:

$$ {}_6C_3 = \frac{6!}{(6-3)!\,3!} = \frac{6\times5\times4}{3\times2} = 20 $$

There are twenty different ways of selecting 3 cards out of six. So you can have twenty different numbers shown within the magic window which shifts with each combination of 3 cards.
Look in the main card and you will find that there are twenty numbers, exactly ${}_6C_3$.

The windows have to be cut on each card so that each combination of 3 cards gives a unique window. This can be done by copying the pattern in the figure or by trial and error.

Mind reader – 2

Materials required: Plain paper and pen.

This is yet another game which looks magical, but has an underlying principle that can be easily grasped. Copy out the tables shown in the figure. That's all that is needed for this game.

$\boxed{1}$	3	5	7	9
11	13	15	17	19
21	23	25	27	29
31	33	35	37	39
41	43	45	47	49

$\boxed{2}$	3	6	7	12
13	16	17	22	23
26	27	32	33	36
37	42	43	46	47
47	43	45	47	49

$\boxed{4}$	5	6	7	14
15	16	17	24	25
26	27	34	35	36
37	44	45	46	47

$\boxed{8}$	9	18	19	28
29	38	39	48	49

$\boxed{10}$	11	12	13	14
15	16	17	18	19

$\boxed{20}$	21	22	23	24
25	26	27	28	29

$\boxed{30}$	31	32	33	34
35	36	37	38	39

$\boxed{40}$	41	42	43	44
45	46	47	48	49

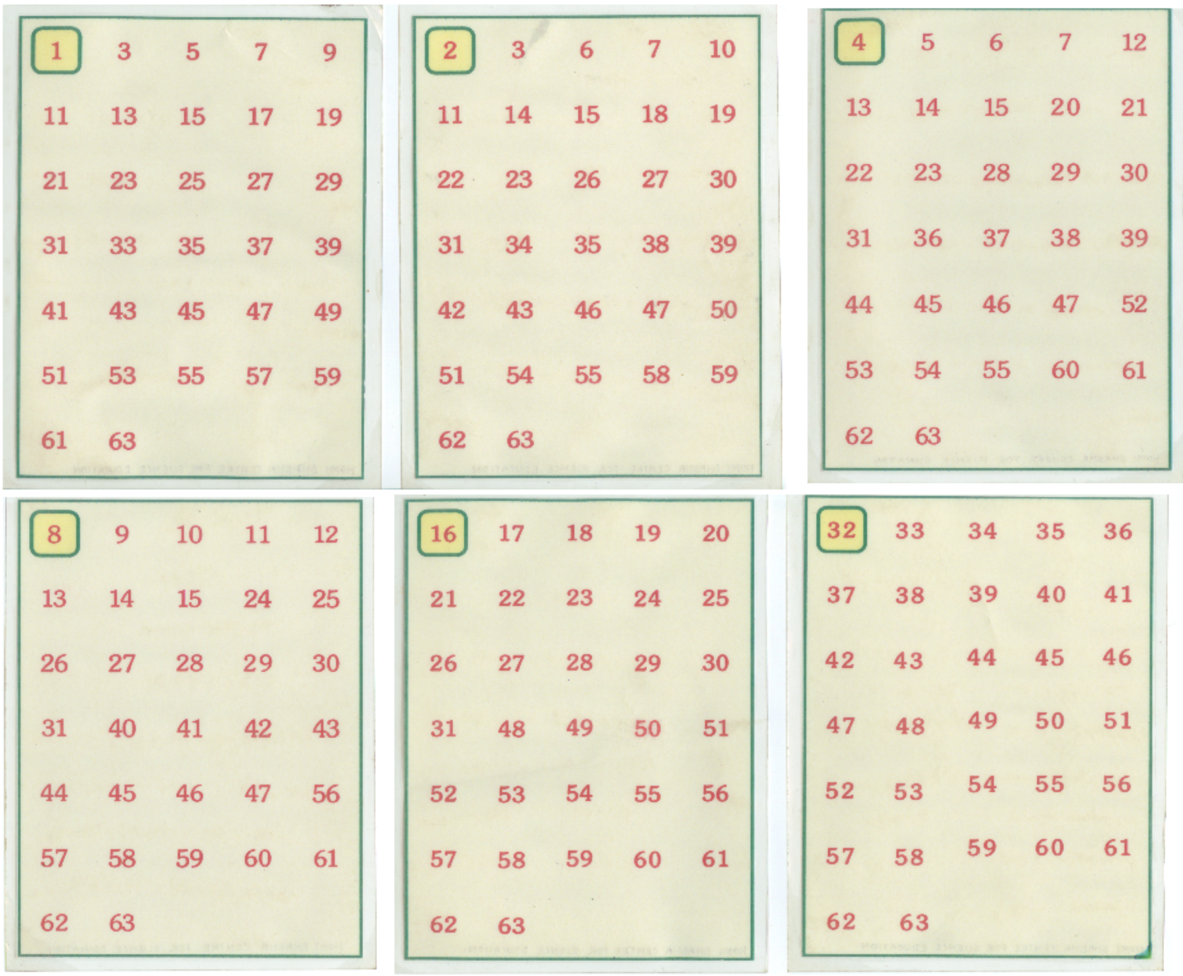
Now ask your friend to think of a number less than fifty. Let him say which of the tables shown in the figure contain the number. Add all the corner numbers in the tables which your friend points out. Their sum gives the number your friend thought of! The principle is so simple that you can make your own tables which are different from these.

To do this first make magic tables for numbers from 1 to 9. You need 4 tables with the corner numbers $\boxed{1}$, $\boxed{2}$, $\boxed{4}$ and $\boxed{8}$. (The box around $\boxed{1}$ means it is a corner number.) You have to put all the numbers from 1 to 9 in these four tables in such a way that adding the corner numbers of the tables in which a number appears gives that number. In other words, you have to write down all the numbers from 1 to 9 as the sum of some or all of the numbers 1, 2, 4 and 8. 1 and 2 already appear in the tables. 3 can be written as 1 + 2. So it must appear in the tables with $\boxed{1}$ and $\boxed{2}$. 4 is already present. 5 can be written as 4 + 1. Hence it must appear in the tables with $\boxed{1}$ and $\boxed{4}$. Similarly continue for all the numbers upto 9.

The number 10 can be put as a corner number in a new table. Now all the numbers from 11 to 19 can be expressed as the sum of 10 and a single digit number. 15, for example, can be expressed as 10 + 5. So place 15 in the tables in which $\boxed{10}$ appears and those in which 5 appears. This will be in the tables with $\boxed{10}$, $\boxed{4}$ and $\boxed{1}$. Continue in this way upto the number 19. For numbers from 20 to 29 make another table and so on upto 49. This is the way in which the tables shown here are constructed.

Now try changing the corner numbers to $\boxed{1}$, $\boxed{2}$, $\boxed{3}$ and $\boxed{5}$. Can you generate another set of tables?

The table shown in the figure is not economical. There is no need to start a new table for the number 10. We can express it as 8 + 2. Hence it can be fitted in the tables with $\boxed{8}$ and $\boxed{2}$. In this way how many numbers can you fit within the first 4 tables? Try to make the most economical set of six tables. Find out the greatest number your mind reader is capable of reading with 6 tables. (It should be able to read any number less than or equal to 63.)



Tic-tac multiply

Materials required: Card paper, pen, buttons in two different colours – about 15-20 in each colour.

This game, which is a modified version of tic-tac-toe (also called dots and crosses) is excellent to sharpen skills of factorization in algebra. It can also be modified to work for younger children and can sharpen single digit or two digit addition and multiplication.

TIC TAC MULTIPLIER

$a^2 + ab$	$4b$	$a^2 - ab$	$a^2 + 2ab + b^2$	-16	$4a + 4b$
a^2	$-b^2$	ab	-4	$-ab$	b^2
$ab - b^2$	$-a$	b	$-a - b$	$-a^2$	$-4a + 4b$
$4a$	$-a^2 + ab$	$-a^2 - ab$	4	$-b^2 - ab$	$-4a$
$4a - 4b$	$-b$	16	$a^2 - b^2$	$-ab + b^2$	$-4b$
$-4a - 4b$	$-a + b$	1	$a^2 - 2ab + b^2$	a	$b^2 + ab$

a b	a	b	4	1	4	b	a	a b
○	○	○	○	○	○	○	○	○

To play the game you need to copy out the board shown here. You also need about 15-20 buttons in two different colours, say red and blue. Each set is given to one player. One red and one blue button are needed for placing on the factor board below the main board, one for each player. The remaining red and blue buttons are for placing on the main game-board.

The game begins with the players placing both the factor board buttons on any two squares on the factor board. The first player to move will move his factor board button (say red) to any of the factor board squares. Now the first player multiplies the numbers under the two buttons on the factor board and places a new red button on the product on the main board. For example, if on the factor board the blue button is on -1 and the red button on 4. The product is -4. So the first player places a red button on -4 in the main game-board. Now it is the second player's turn. She too first moves her factor board button, the blue one in this case. She then multiplies the numbers under the two factor board buttons and places a blue button on the product in the main board.

On the factor board the first player can move only the red button, the second player can move only the blue one.

If a player makes a mistake while multiplying, he or she loses a turn. The first person to get 4 small buttons in a line wins.

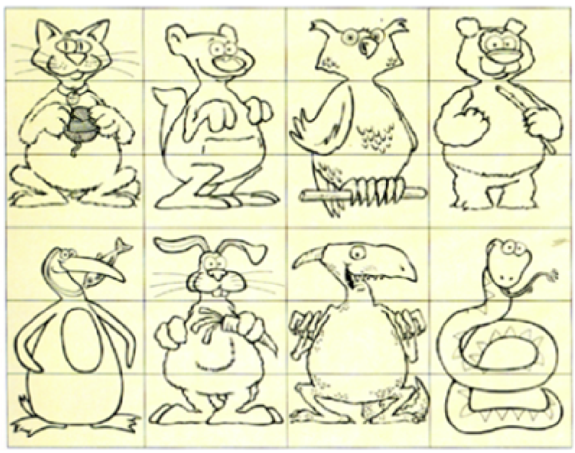
This game is a constrained version of tic-tac-toe. In ordinary tic-tac-toe, buttons (or dots and crosses) can be placed anywhere on the board. In the constrained version, the factor board limits the number of squares where it is possible to place the button. By varying the type of constraint, one can modify the game to focus on a range of mathematical skills.

A combination book

Materials required: Eight sheets of paper

This is a picture book containing eight pictures of different animals. Each page is cut into three parts, exactly dividing each picture of an animal into the head, body and feet. The book can be prepared by carefully drawing the pictures given here. The pages can then be stapled together such

DESIGN FOR COMBINATION BOOK



that the dotted lines in each page are aligned. All the pages can then be cut into three parts along the dotted line.

Now heads, bodies and feet from different animals can be combined to form delightful pictures of strange animals. How many pictures are possible in all? (By keeping a head and a body constant and varying the feet, one can obtain 8 pictures. Now by keeping the head constant and varying the body and the feet, one obtains $8 \times 8 = 64$ pictures. By also varying the head one obtains a total of 512 pictures.)

Dominoes

Materials required: Chart paper, paper clips, pen.

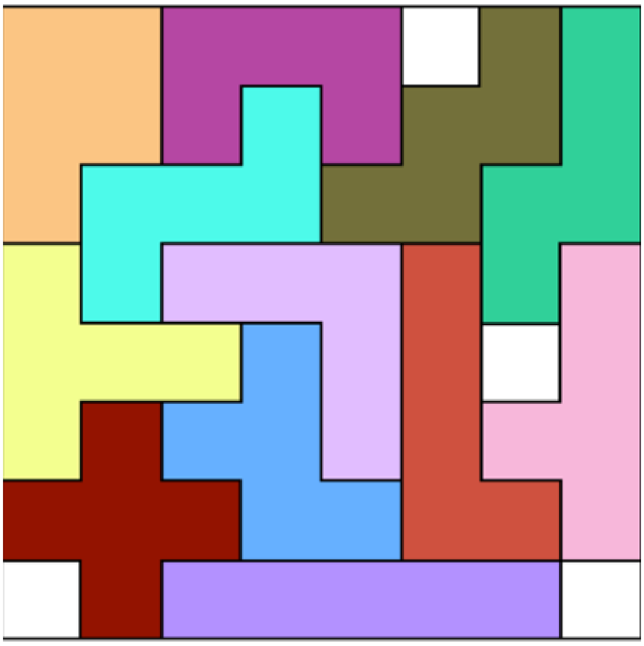
Dominoes are blocks made up of two squares. The activity consists of filling up a given square with dominoes. A chessboard is put before the students and they are asked how many dominoes are required to cover the chessboard. Now two opposite corners of the chess board are covered with black paper. How many dominoes are now needed to cover the chessboard? Ask the students to try and cover the chessboard with dominoes which have been cut out beforehand.

A square of 6×6 and another of 4×4 are available for the students to try out a similar exercise with them. Ask the students to reason out why it is impossible to cover the boards when two opposite squares are blanked out.

Pentaminoes

Materials required: Chart paper, paper clips, pen.

PENTAMINOES FROM A 8 × 8 BOARD



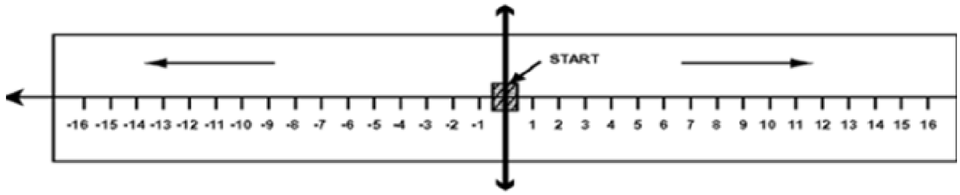
Pentaminoes are made by joining five squares so that at the joints, one side of a square fully joins with one side of another. There are twelve possible arrangements in which five squares can thus be joined to each other. Hence there are 12 possible pentaminoes.

Ask the students to find out all the possible pentaminoes. Let them cut out these pentaminoes from card paper. The total area of the pentaminoes, assuming each small square from which they are made up to be of unit area is $12 \times 5 = 60$ square units. Give the students a 8×8 board which contains 64 squares. Can the square board be covered with all the twelve pentaminoes, so that four squares remain. Although there are many possible ways of doing this, it may take a while to find even one arrangements which fills 60 of the 64 squares. One of these arrangements is shown in the figure. Some of these possible arrangements leave four squares exactly in the middle. Can the students find such solutions?

Number line checkers

Materials required: Chart paper or cardboard, pen, buttons in two different colours – 5 buttons in each colour.

Here's a simple and delightful game which sharpens skills at adding and subtracting signed numbers. This is suitable for children in primary school.



Copy out the number line given in the figure to a convenient size. Two players play the game at a time. They get 5 buttons each in different colours, say red buttons for the first player and blue buttons for the second.

In the beginning of the game all the buttons are placed at the starting point which is zero.

Now the first player picks a card. There is a number written on it. He moves one of the red buttons to this number on the number line. Now it is the second player's turn. She too picks up a card and a blue button on the number line on top of the number which appears in her card.

Every time a player picks a card he or she can move a fresh red button to the number on the card or can also move a button already on the number line by the number of places shown on the card. For example, if there is already a button on -5 and the card shows -7, the player can move the button to -12.

The aim of each player is to capture as many buttons as possible. Both red and blue buttons can be captured by each player.

A player can capture buttons in two ways:

- A player's button moves past -19 0r +19. Then he or she win the button.
- One player's button jumps over the second player's button. Then the second player's button is 'killed' and the first player gets to keep it.

The game stops when either all red buttons or all blue buttons have left the number line. The person with the maximum number of buttons with her or him (both red and blue) wins. Some additional rules:

- Two buttons cannot occupy the same position on the number line.
- If a player makes a mistake in arithmetic while moving the button, the button goes back to the starting point.
- A player can jump over his or her own button. No problem. The button stays alive.
- If a player jumps over two (or even three) of the other player's buttons, he or she gets to keep all of them!

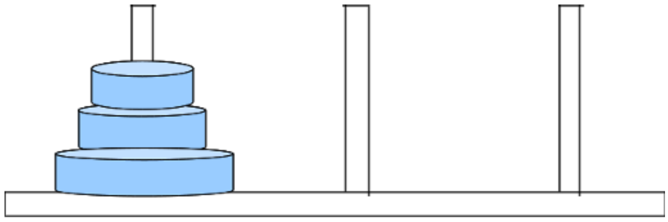
Tower of Hanoi

Materials required: Board with three rods, discs.

How to play:

We have a board with three rods. In one of the rods. We insert several discs arranged in order pf magnitude with the largest at bottom. The task is to transfer all the discs from the first rod to one of the others in such a way that the final arrangement is the same as the original one. The rules are, only one discs can be moved at a time. No disc should be ever placed on the top of a disc smaller than itself. You can make use of one free rod while transferring.

Start with just two discs and count the minimum number of moves required to transfer the discs to one of the other rods using the third rod, then try it with three discs, 4 discs and so on. The aim is to find a rule which connects the number of moves with the number of discs.



Answer: The minimum number of moves needed to transfer all the discs from one rod to the other rod in the required order is 2^n-1, where n is the number of discs. For example, to transfer 5 discs you would need at least 31 moves.
Recursive relation:

Number of discs	Minimum number of steps required to transfer discs
1	3
2	5
3	7
4	15

When you want to shift all the 3 discs from one pillar to another, first you need to shift the two discs on the top to another pillar. Both of them have to be on the same pillar as one pillar has the largest disc and the third pillar has to free for shifting the largest disc. Now you have seen that the no. of steps needed to shift two discs is 3. So you will need 3 steps, then you will shift the largest disc on the empty pillar. And then you have to shift the remaining two on top of the largest disc, this needs another 3 steps. Hence the total no. of steps needed are 3 + 1 + 3.
Try to see this relation yourself using 4 discs.

Number of steps required to transfer 2 discs =2 (Number of steps required to transfer 1 disc +1)
Number of steps required to transfer 3 discs = 2(Number of steps required to transfer 1 disc +1)
Number of steps required to transfer n discs = 2(Number of steps required to transfer (n-1) disc +1)

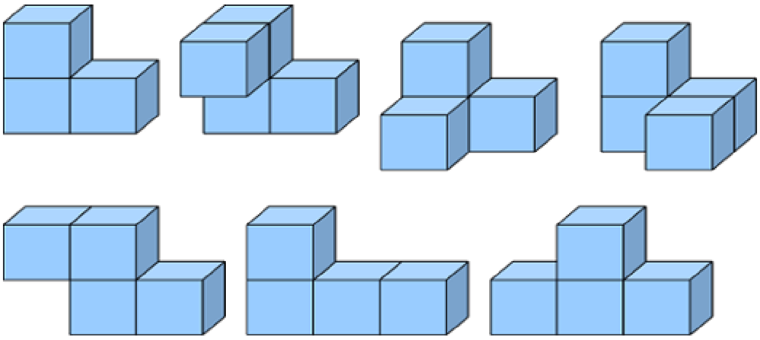
From the above table we can see that

Number of steps required to transfer n discs = 2^n -1

Number of steps required to transfer (n+1) discs = $2(2^n -1) +1= 2^{n+1}-1$

Soma's Cube

Material: 7 pieces as shown below.



How to play:
Assemble these seven shapes to form a big cube (3×3×3)

there are about 220 ways of making this cube. Try and discover as many ways as you can. The seven pieces of Soma's Cube Puzzle can also be arranged to make a cot, a chair, a snake, etc. You can arrange them in different ways to make your own design.

List of sources

Some of the activities, games and puzzles described here are taken from the following sources and modified or developed:

1. The Mathematics Teacher, Vols 84-88, 1992-1996.
2. How Mathematics Works, by Carol Vorderman, Dorling Kindersley, London, 1996.