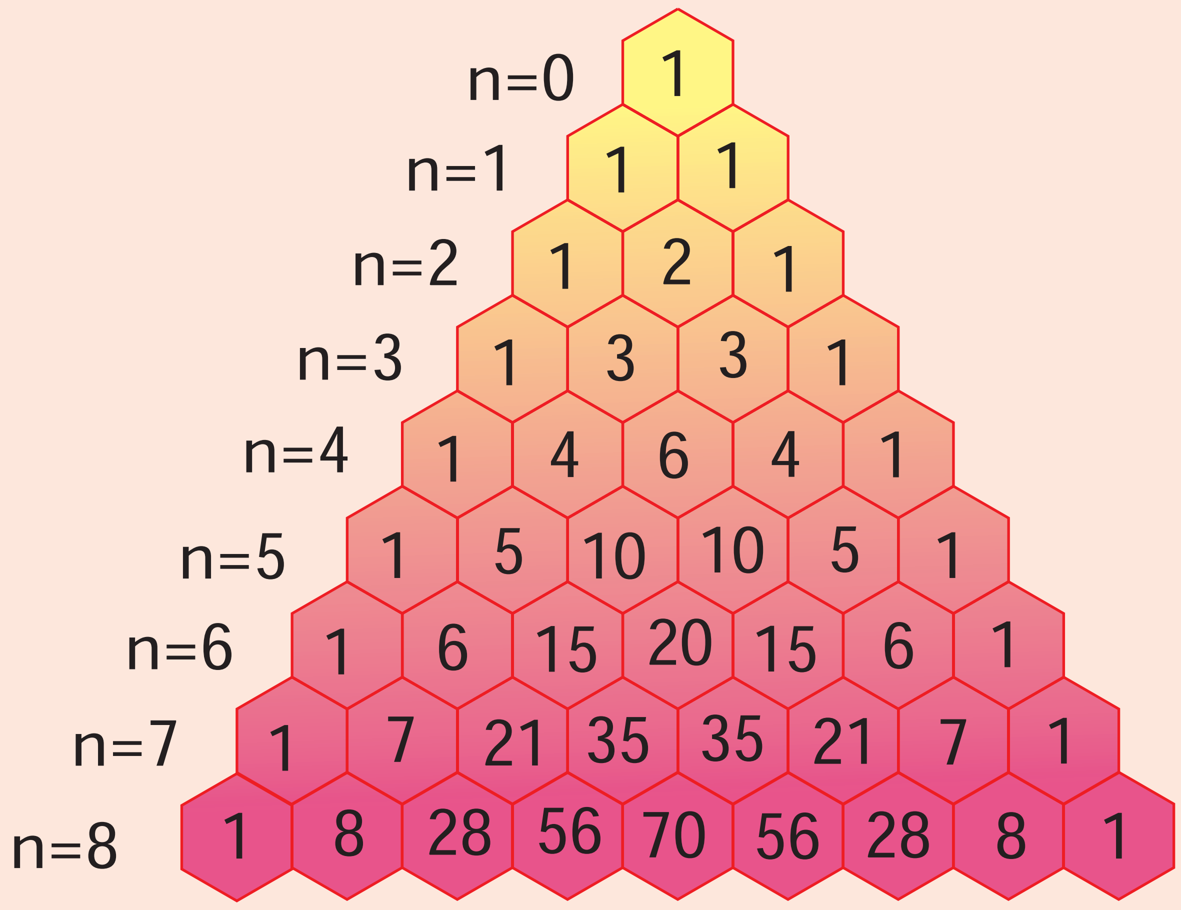
Pascal's Triangle



The Pascal's triangle contains many interesting patterns.

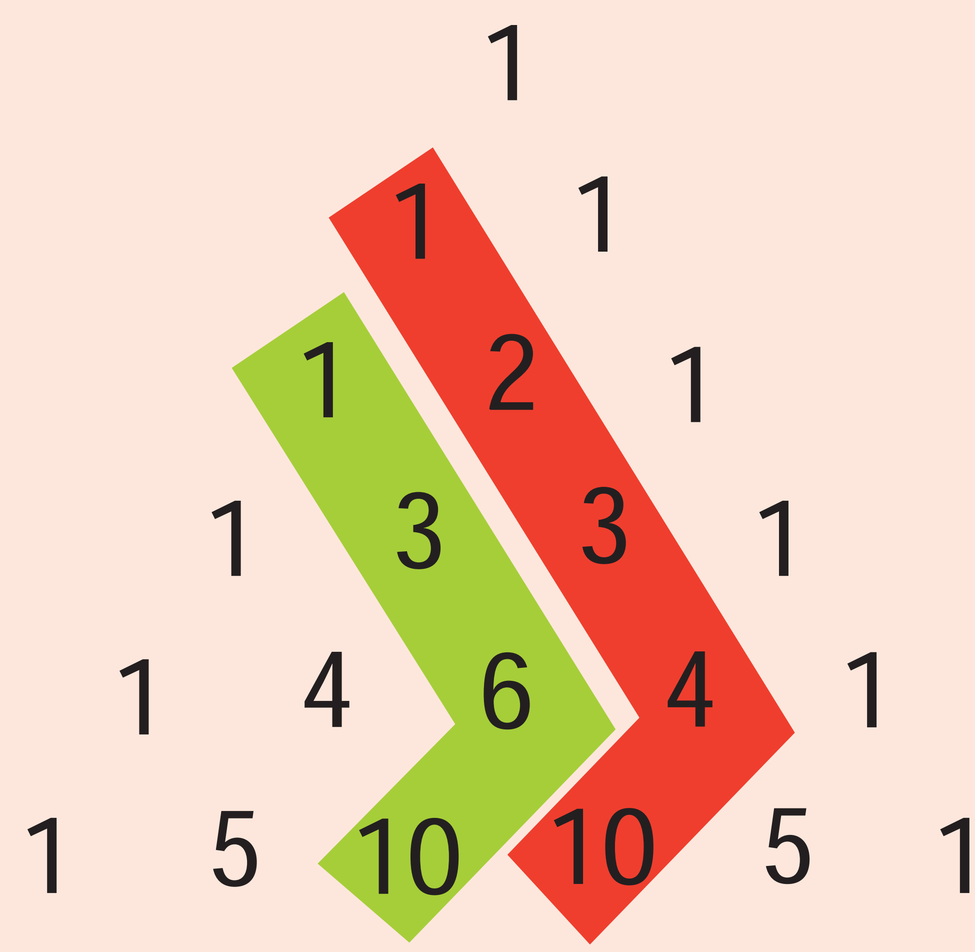
To construct the Pascal's Triangle start with '1' for the top (i.e., the 0^{th}) row . The beginning and end of every subsequent row is also '1'. For all other positions, write down the sum of the two numbers immediately to the left and right in the previous row.

- The n^{th} row gives all the coefficients of the binomial expansion $(x+y)^n$.
  For example, the 3^{rd} row contains the coefficients 1,3,3,1 corresponding to $(x+y)^3=x^3+3x^2y+3xy^2+y^3$.

- The sum of the digits in the n^{th} row equals 2^n.
  For example, in the 3rd row, 1+3+3+1 = 8 = 23.

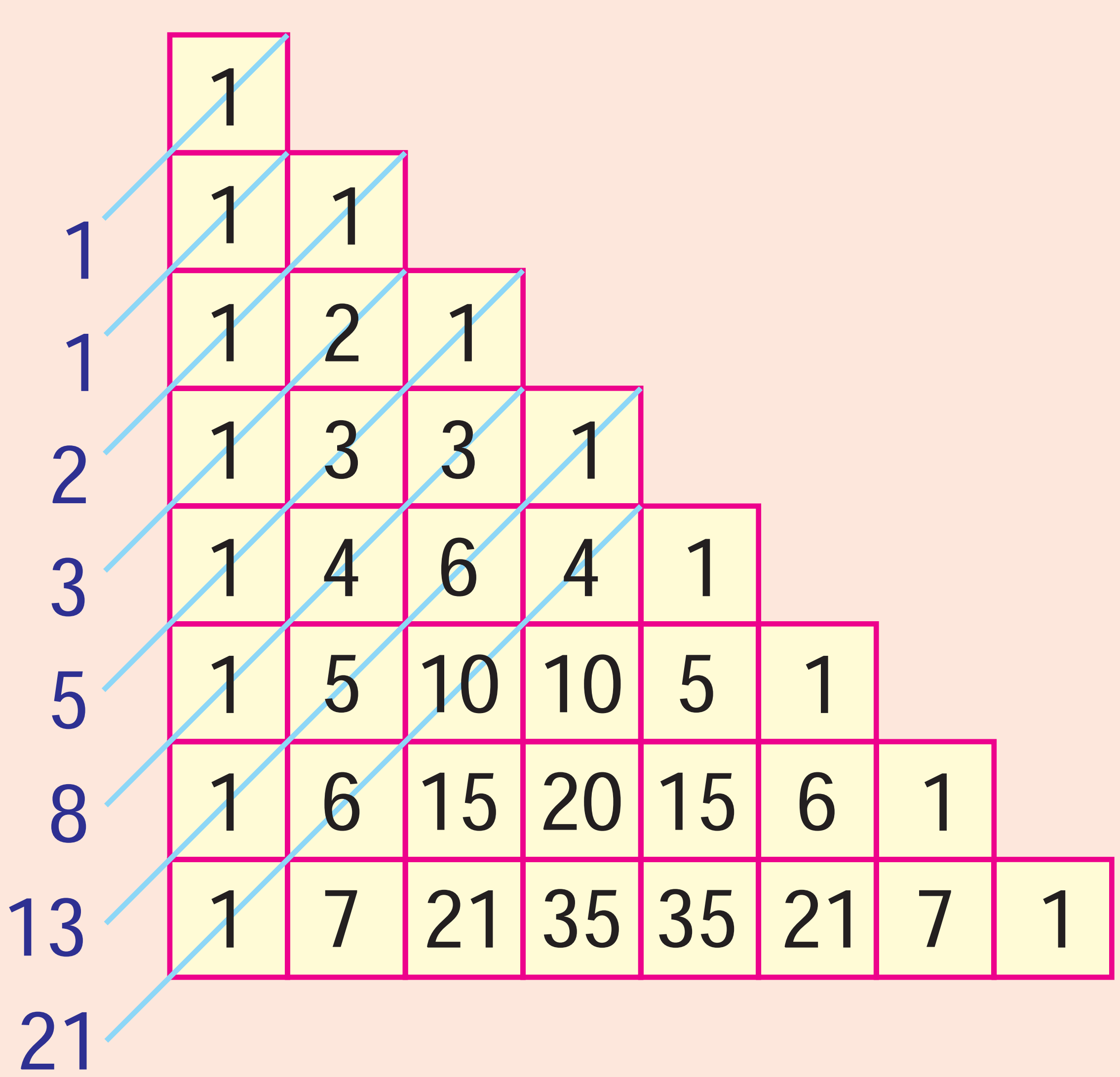
- The r^{th} number in the n^{th} row gives the number of different ways in which 'r' objects can be selected out of 'n' objects ($_nC_r$).

- If 'n' is a prime number, all the numbers other than '1' in the n^{th} row are multiples of 'n'.
  For example, in the 7th row, 7, 21, 35 are multiples of 7.



- The diagonal lines of the Pascal triangle contain several patterns. Each number other than '1' is equal to the sum of all the numbers in the preceding diagonal line above that number.
  For example, 10 = 1 + 3 + 6 (green diagonal) and 10 = 1+ 2 + 3 + 4 (red diagonal).

- Starting from the right end of any row, take the digit in the units place to form a multi-digit number. (The digits in the non-unit places must be carried over to the next number). The number so formed is equal to 11^n.
  For example, in the 5th row,161051 = 115.



- Arrange the numbers as a right angled triangle. The sequence of numbers arrived at by adding the numbers diagonally form the Fibonacci sequence: 1,1, 2, 3, 5, 8, 13, 21,…. where each number is the sum of preceding two numbers.

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