Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING

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The Journal of Mathematics Education at Teachers College is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

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This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue's guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form.* Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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Reformed Curriculum Framework: Insights from Teachers' Perspectives

Shikha Takker

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The National Curriculum Framework 2005 envisions a reformed curriculum that connects knowledge and learning to the child's context. While concerns of equity, accessibility, and social transformation have been aimed through education, the crucial link of 'teacher'—instrumental in realizing these aims—seems missing. This study explores teachers' views about mathematics, their understanding of children's roles and issues of pedagogy within the context of the curriculum document. The paper is an attempt to bring forth the teachers' perspective on a reformed curricula, the kind of constraints it poses upon them in practice and the lack of spaces for teachers to critically engage with the curriculum. Considering classrooms as rich resource for teacher learning 'from' practice, teachers need support structures and forums to discuss, evaluate, and enrich the curricular suggestions for reforms and critically inform practice.

Introduction

During the past two decades, Indian education has gone through major changes in the teaching-learning process as well as the structure of education institutions (Ganesh, 2005). The National Council of Educational Research and Training (NCERT) is an apex body entrusted with the responsibility of developing and reviewing curriculum framework for India. The curriculum document provides guidelines for making syllabi and textbooks for grades I to XII of formal schooling. The National Curriculum Framework (NCF) proposed in 2005 focuses on many inter-related aspects of education, namely, the aims of education, the social milieu of children, the nature of knowledge in its broader sense, the nature of human development, and the process of human learning (NCF, 2005, p. 4). Two significant attributes contributing to the distinctive approach (different from earlier curriculum frameworks) in designing NCF 2005 are: first, a democratic process of engaging stakeholders such as curriculum designers, researchers, educators and practitioners from different parts of India to collaboratively work toward producing an effective framework for action (elaborately discussed in Sadgopal, 2005); and second, it is a conscious effort of making education equitable, accessible, and more meaningful for the learner (NCF, 2005, p. 7).

NCF 2005, with its different approach and emphasis, calls for reforms in school education that demand a shift from traditional instructional practices to those engaging learners in meaningful activities. There is an attempt to rejuvenate the central place of children in the spectrum of teaching, learning, and schooling. Especially in the case of mathematics, NCF 2005 strives to analyse the reasons for the fear and failure that children face when doing mathematics and encourages appropriate measures to help circumvent the problem.

Understanding of Mathematics Teaching and Learning

NCF 2005 proposes that the aim of mathematics teaching and learning is mathematisation of the child's thought processes. Instead of mathematics learning being loaded with content, the emphasis is to learn meaningful mathematics, and to focus on developing the inner resources of the child to bring clarity of thought while pursuing logical conclusions with an ability to handle abstractions. Doing mathematics, in accordance with NCF 2005, reflects a shift from more external aims to doing mathematics as a worthwhile activity in its own sense. Philosophically, this represents a teleological orientation towards learning mathematics rather than merely an instrumental one (Carr, 2003).

An informed learning perspective rooted in sociocultural and cognitive paradigms of learning is also reflected in the document. For instance, the view of children learning mathematics envisioned is that they learn to enjoy and talk about mathematics, pose and solve meaningful problems, use abstractions to perceive relationships and structure, with an underlying belief that all children *can* and *need* to learn mathematics (NFG. 2006a). In the Indian context, a majority of students fear mathematics because of the failure they experience and a lack of exposure to the beauty and abstractions of the subject. The active urge of a learner to construct, deconstruct and reconstruct is an important part of his/her process of learning to discover. And this is what needs to be elicited through the teaching and learning of mathematics.

Besides, there is also a concern towards *children's view of mathematics*. Alienation of the curriculum and ineffective assessment methods increase the disconnect between student and mathematics. The cognitive inappropriateness and push to rote memorize and reproduce knowledge further accentuates the misery of a child. NCF (2005) proposes an

activity-based orientation of the curriculum, making use of concrete manipulative to facilitate the formation of mental models and development of abstract ideas in children. This will help learners develop interest in mathematics and provide motivation for making connections between different knowledge forms and their application. A cognitively appropriate curriculum and learner-sensitive pedagogy (NFG, 2006a) that emphasizes, for example, the relation of mathematical notions in classrooms with reallife contexts and systematically introduces the ideas of numbers, space, an eye for quantification, visualising patterns, estimation, etc. is central to facilitating such an enriched learning interaction.

Such concerns about mathematics learning and its pedagogy are central to NCF 2005. It is important, therefore, 'to understand how teachers interpret these issues and bring them to use in their everyday classroom because they are transactors of this curriculum in practice' (Lester, 2007). In fact, in relation to reforms and teachers' perspectives, mathematics educators such as Shulman (2004) and Ball (1990) have argued that any promising reform that envisions an impact on education needs to understand teachers' views about the subject, their understanding of children's thinking, as well as their own beliefs about the process of teaching.

Context of the Study

NCF 2005 with its valued focus on the child and learning has strong implications for education. However, it poses equally strong demands on teachers. An effort to address the demands on teachers and their preparation has been a part of the deliberation in designing NCF 2005. This is presented in the form of a national focus group report on teacher education (NFG, 2005b). From the viewpoint of practice, it is important to study teachers' perspectives about the proposed changes (stated curriculum) and how they get translated in the classroom (enacted curriculum). This paper reports the voice of teachers and gives their perspectives on the reformations introduced under NCF 2005, with a focus on mathematics teaching and learning. The study is grounded in a belief that any curriculum reform endeavour is incomplete without teachers' voices, as they occupy an active role in enacting the curriculum. Some related concerns like teacher professionalism and social scientific issues are also important but they are not directly within the purview of this study and hence are not a part of this paper.

Objectives of the study

The study aims to understand the potential gaps teachers face in translating ideas in NCF 2005 to classroom practice by

- Probing teachers' perceptions of teaching and learning mathematics with reference to National Curriculum Framework 2005.
- Observing classroom dynamics of an exemplar classroom (driven by the approach proposed by NCF) and a non-exemplar classroom.
- Identifying the struggles (problems—anticipated/ faced) that teachers encounter in the process of implementing the new curriculum and reasons for the same.

Methodology

The exploratory study reported here probed teachers' perceptions on some of the propositions related to mathematics teaching-learning, suggested by NCF 2005, using a questionnaire as a research instrument. This was followed by in-depth observations from two teachers' classrooms regarding a particular mathematical concept, i.e. number operations. The researcher briefly interacted with the teachers after each classroom observation to understand reasons for the decisions they made while teaching. The theoretical framework for development of questionnaire and analysis draws from the work of Ernest (1991) on teacher beliefs. He proposed three components which are central to understanding teacher beliefs: the teacher's view or conception of mathematics as a subject of study, the teacher's view on the nature of mathematics teaching, and the teacher's model or view of the *process* of learning mathematics.

Sample

The considerations that guided the selection of the sample for this study were: familiarity of respondents to NCF 2005, and a representation of different kinds of schools (government, government-aided, private, and alternative schools) in Delhi. Fifteen elementary school mathematics teachers (grades I to VI) who informed the researcher that they were aware of NCF 2005, either through their in-service or pre-service program and had discussed it with their colleagues, volunteered to participate in the study. A conscious effort to include teachers who were familiar with NCF 2005 ensured that they were able to identify and relate with the items in the questionnaire and also the pedagogical tenets included in the document.

Sources and Tools for Data Collection

The study was conducted in 2 phases. The first phase involved a survey to probe teachers' perceptions of teaching and learning mathematics in classroom contexts using a questionnaire. The questionnaire developed included statements from NCF 2005 concerning epistemological notions about mathematics, children's understanding of mathematics, and aspects of pedagogy. The questionnaire was content validated by mathematics educators, who were also a part of the curriculum designing committee, ensuring that the propositions used were authentic and appropriately presented. Reliability was assured in terms of parallels with the propositions in the document and validity in terms of suitability of issues addressed, sequence of items, appropriateness of language and issues related to comprehension of statements. The framework for analysis emerges from literature and focuses on three crucial aspects of mathematics teachinglearning, namely: teachers' perception of mathematics, their understanding of children learning mathematics, and their stance on pedagogy of mathematics. Questions involved a variety of formats: open-ended, semantic differential, and agree/disagree statements requesting reasons for choice. The questions expected teachers to answer about their beliefs and practices on mathematics teaching and learning in classrooms.

Responses of teachers to the questionnaire were analyzed. Based on the pattern of responses, two teachers were identified, one (referred to with psuedonym P) who seemed convinced vet confused regarding ideas posited in the NCF 2005, and the other (referred to with psuedonym Q) who was skeptical about the consequences of it. For practical reasons of access and feasibility, two participants that had distinct perceptions about NCF 2005 were selected for the second phase. The second phase of classroom observations aimed to study how aspects enshrined in NCF 2005 are reflected in mathematics teacher's practices. The researcher observed teachers for the contact sessions involving "number operation" taught to grade IV students after seeking their consent. A brief interaction with the teachers after each class helped the researcher to probe informally teacher's ideas and issues related to practice.

Analysis

In contrast to a quantitative mode of analysis, this study used a qualitative methodology to bring out the richness and coherence in the two components, namely; the survey, and classroom observations supported with findings from brief interactions with teachers. The first part of the analysis discusses insights from the survey that revealed **teachers' perceptions about mathematics, notions of children learning mathematics, and ideas concerning mathematics pedagogy in relation to NCF 2005.** Further, a conscious effort has been made to analyze whether teachers' responses to items in the questionnaire relate to their classroom practices. The second part of analysis concerns insights from classroom observations of two teachers, selected on the basis of criteria mentioned above. Insights from the analysis of classroom observations suggest a mismatch between teachers' understanding of NCF and their actual practices. These are discussed in a separate section.

Teachers' Perceptions about Mathematics

One of the initial questions was a semantic differential kind and probed teachers' beliefs about mathematics. It was interesting to note that a majority of teachers (10 out of 15) felt that mathematics is about reasoning, application of techniques and solving challenging problems. This observation is corroborated by a unanimous response to an agree/disagree question by all teachers who agreed that there is more than one way to learn mathematics and that students should learn mathematics through both problem solving and textbooks. Teachers' strong belief in problemsolving approach is a surprising deviant from the common perception of mathematics among students of it being dry, algorithmic and abstract (NFG, 2006a). Interestingly, a few teachers (5 out of 15) who believed that mathematics is about application also asserted, through their response to another item, that mathematics is distinct and not related to other disciplines. Application of mathematics for these respondents seems to be limited in scope to the domain of mathematics itself, suggesting a very narrow understanding. A large proportion of teachers (11 out of 15) consider mathematics as certain knowledge with little scope for doubt or ambiguity. Furthering this trend, the responses of slightly more than half of the teachers (8 out of 15) demonstrated a belief that mathematics is about absolute knowledge. Implicit in such a response trend is the general conviction that mathematics is a complete or fixed body of knowledge. Such a perception has important implications for teaching-learning from an epistemological standpoint.

Children Learning Mathematics

Most (11 out of 15) teachers believed that learning mathematics requires both practice as well as insight. While teachers acknowledged the significance of teacher and students efforts as constituents of good mathematics teaching, they differed on the conditions that are required for mathematics learning. Slightly more than half (8 out of 15) of the teachers believed that mathematics requires understanding in contrast to memorization. However, another question requesting a response in agree/disagree format elicited a divided opinion, with similar proportion of teachers (8 out of 15) disagreeing that memorization and performance of algorithms are in themselves not important to learning mathematics. Such a pattern reveals a confused take of the same proportion of teachers on the role of memorization in learning mathematics. A substantial proportion of teachers (6 out of 15) believe that the onus for learning mathematics jointly lies on strong students and good teachers. However, it is surprising to note that an equal proportion felt that the onus rests solely on good teachers. The central role of teachers in learning mathematics is as prominent as joint roles of teacher and student in taking the responsibility of learning. The divided opinion is a cause of concern.

Pedagogy of Mathematics

A large majority of teachers (13 out of 15) agreed that children's conversations are indicators of their understanding. A concord in teachers' responses is evident in the disagreement by a similar proportion of teachers to the statement, "children learn by listening to the teacher and not by talking to each other." Similarly, on a different item concerning their view of pedagogy, 12 out of 15 teachers disagreed with the statement that, "A child answering the questions raised by the teacher is more important than their posing problems." Although about a third of the sample (5 out of 15) believed students' questions during teaching distract, the rest disagreed with the statement. A few reasons suggested for the disagreement were that questions arising in relation to the lesson can orient the class discourse; two of the teachers mentioned that such questions may give cues about the thought processes of children.

Teachers, as evident in their responses, maintained that an understanding of the relevance of mathematics in daily lives is more important than enjoying mathematics. This is particularly relevant in the context of teaching. This is, however, different from the report which states that mathematics with prevalence to the logic of the discipline than the psychology of its learning as the basis of curriculum makes it acquire the image of an esoteric discipline which has little application in the real life of children (MHRD, 2008).

A large majority of teachers (13 out of 15) felt that lessons must include a wide variety of mathematical tasks for maintaining the learner's interest. Though teachers acknowledged this fact, they perceived it as difficult in practice given the structure of the required planning and time frame. Having flexibility in lessons was perceived by teachers (9 out of 15) to be a crucial factor for good mathematics teaching; however, it was difficult for them to articulate precisely what that flexibility means in the classroom. A similar proportion also expressed that they need to both help students develop a liking towards mathematics and help them see mathematics as useful. This reflects how teachers perceive mathematics as a domain of human understanding. The rationale for establishing a liking for mathematics comes from their reference to the NCF 2005. But, nowhere did the teachers mention that there is a need to relate children's thinking and their beliefs to inform mathematics pedagogy which is a crucial component of NCF.

Mismatch in Teachers' Understanding and Their Practices

It is evident from the analysis that at least 2 levels of complexities operate in practising teachers' implementation of the NCF 2005. On one plane, there is a mismatch of teachers' use of terminologies suggested in the document and how they translate them in their pedagogic practices. The second plane involves their own understanding developing over a period of time not seen in this way by the teachers. The two planes of complexity often interact as was evident in some of the examples from classroom observations.

One of the evident changes in teacher practice is noticed in pedagogy where teachers acknowledge to have made changes in their teaching strategies. In the two cases observed, teacher P encourages a lot of mathematical discussion but often does not know after a point which direction should it be oriented to. Teacher Q is confident in encouraging a lot of practice in her classroom (which is a pedantic routine of giving a lot of numerical problems for children to solve). Both teachers shared that they do so because they believe that such a strategy will benefit their students and has been explicated in the curriculum document. On probing further it was difficult for either of them to rationalize their decisions. The teachers' stance is 'selective' and indicates that with their interpretative understanding of their role in classroom context, there is little deliberation on the value or feasibility of their actions.

Another example is embedded in the semantic context of the document. The sense of 'meaning making by students' as implied in the document is quite varied from the idiosyncratic and self-made understanding by teachers posing several problems in their practices. A few examples of certain notions used in the document include flexible lessons, child-centrism, mental models, conceptual understanding, and activity-based learning. The two teachers, for instance, interpreted the activity-based classroom differently. Teacher P considered it as engaging children in play using materials and objects, but often had questions on how to make a transition from concrete to abstract, while teacher Q viewed it as paper folding, cutting activities, etc. in geometry lessons. She also conceived it as children listening to her attentively at the introduction of an activity. Such a myriad of semantic interpretations by different teachers enters into classroom unintended and without reflection on the consequences of teaching and learning.

The inconsistency in teachers' responses may plausibly be a result of a juxtaposition of their experiences with an incomplete or "fashionable" understanding of ideas in the NCF 2005. This can be evidenced by the variety of reasons explicated by teachers (to the questionnaire) for "agreeing" to the statement, "Talking in a mathematics classroom is important." The variations included conditional situations, such as *only* at an introductory stage, *only* to discuss in order to solve problems, and *to the extent* that it can be bound with teachers' directions [teacher hegemony]. Another class of reasons were purpose-oriented claims such as, discussion with hands-on experience and concrete life contexts is important, and talking helps to know the level of understanding. Another personal preference was also elicited in the response of a teacher who said, "[Talking in the classroom] is important but I don't like a noisy classroom."

Teachers' responses to the questionnaire elicited varied (mis)interpretations to the propositions of the NCF 2005, which has serious implications for teaching and learning of mathematics. Teachers seem to struggle with internalizing ideas found in the NCF 2005 and in the process of an emerging understanding, coupled with a tendency to keep up with the trend, opt for an interpretative version. Many of the teachers expressed the lack of spaces where they can discuss issues related to mathematics teaching-learning without being evaluated for their worth.

Conclusion

The paper reports an empirical investigation that blends survey and classroom observations of elementary school mathematics teachers to investigate the mismatch between ideas enshrined in National Curriculum Framework 2005 (proposed curriculum) and how they get translated in classrooms (enacted curriculum). The exploratory study was conducted in two phases: a survey, followed by classroom observations of two teachers. The findings of the study indicate that teachers are often confused about how theoretical ideas of reform should be interpreted and implemented in the classroom.

Teachers, like learners, interpret the phrases or propositions of a curricular document in ways, which are not in line with the reformed ideas presented in the document. For instance, the significance of encouraging talk in mathematics classroom as envisaged by the document is for the purpose of helping children make meaningful connections. However, teachers see the possibility of talk in a classroom only under certain conditions. Similarly, the activity-based didactic orientation of the curriculum has been understood by teachers as a pedagogical tool to create interest in students at occasional intervals, thus understanding activity in a restrictive sense. There seem to be a gap that exists between the stated approach and what counts as a pedagogic strategy for teachers. Although teachers are convinced with pedagogic strategies suggested in the document, they find it difficult to realize it in practice. For instance, the document suggests a progression from the concrete to the abstract. Teachers although being supportive of this pedagogic move, find it difficult to operationalise and rationalize it in their practice.

Teachers, during their interviews, explicated the need for spaces for sharing and discussing their problems and struggles. There are rare opportunities for them to discuss and cognitively engage with challenges arising out of the practices informed by ideas in the document, resulting in ineffective outcomes. Crucial to our understanding is the awareness of teacher's reality that they operate within structural constraints of school systems, often tightly framed and insulated. In a subtle sense, there seem to be a lot of expectations placed onto teachers, not just being aware of NCF 2005 but also being capable of implementing the curricular ideals independently, i.e. without any orientation to the philosophical shift of reforms. Also, the curriculum document in itself speaks little about crucial aspects like the need for understanding teachers with respect to their academic and contextual backgrounds, their profound understanding of the content, the structures in which they operate, the potentials for presenting scaffolds for understanding, which are indispensable to the transaction of curriculum in practice. The undervaluing of teacher's role in enacting any curriculum creates confusion, causing an impediment to the implementation of desired reforms.

The chasm between principles, values, and suggestions in the document, and teachers' own practices and their interpretations is wide. The hope of revitalising school education in India, via an idealistic or ideologically driven attempt at revising curriculum, will probably meet with little success if the central agency of the teacher remains unrecognized (NFG, 2006b). To engage with teachers would mean a serious involvement in the curriculum and seeing them in light of the contextual process of teaching and learning. We need a radical shift from looking at teachers as 'implementers of a curriculum' to one that views them as empowered individuals. informed by the curriculum, taking their decisions into the classroom. The task is not easy in the developing world, as Hargeaves and Lo (2000) point out, since there is also a tendency to see teachers as 'implementing policy', and researches have noted a reluctance among teachers to embrace those efforts that aim to improve their quality. Other important concerns include issues of teachers' selfidentity as professionals, their training, working conditions, relationship to administrators and functionaries of State, and institutions in charge of curriculum and textbook design (Kumar and Sarangapani, 2004). However, a culture where teachers are empowered and engaged into issues pertaining to education can be aimed at with carefully designed pre- and in-service programs where teachers are given opportunities to create shared understanding and discuss issues central to the practice of curricular ideas. There is a need to emphasise the proposed

as well as the enacted curriculum for any reform to effectively translate in practice.

Future Directions

The NCF 2005 document does not pertain to a single subject but encompasses a broader perspective of looking at schooling and education through the lens of construction and reconstruction of experiences, a form much different from the didactic modes of teaching practised in most Indian schools. Also, the document emphasizes the learning environment and child as an 'active' learner but the role of teachers in the process of learning is undervalued. Will a teacher, who is constantly questioned in relation to the kind of a classroom s/he wants, be in a position to make informed decisions about the long term objectives of mathematics teaching and learning outlined in the curricular framework? Any attempt to encourage teachers to modify their beliefs about teaching and learning must entail the explicit recognition of existing beliefs and their situated nature, attend to the constraints and difficulties under which teachers work, and allow teachers time to reflect on the contradictions between 'good' practice and their 'existing' practice (Swan, 2006). There is a need to investigate deeply into the complexities of classroom practices to understand the nature of support that teachers need. As Batra (2005) suggests, the creation of structural spaces within teacher education institutions and the convergence of institutional linkages can provide the opportunity to bridge the divide created between the school curriculum and the teacher. Tasks that engage teachers to articulate, discuss and evaluate the curricular propositions vis-a-vis their practices, have to be designed and implemented. If the aim is to create teachers who are reflective practitioners and not merely implementers of the curriculum, insights from classrooms need to be utilised for further reformations in curricula.

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