Homi Bhabha Centre for Science Education
Tata Institute of Fundamental Research

Mathematics Activity Manual
Prepared as a Part of an Internship Project

Prepared
by
Ekta Shokeen

Edited
By
Shweta Naik

Table of Contents

Transforming Star

Objectives:

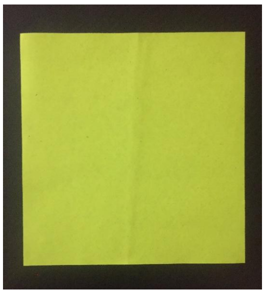
- To appreciate the use mathematics in making origami star
- To enhance the observation, visualization and spatial reasoning skills

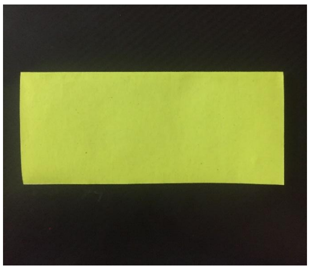
Introduction:

Origami is a type of art first originated in Japan. It is possible to fold many beautiful shapes in origami. Most amazingly, many astonishing pieces of origami are produced from a single piece of paper, with no cuttings. Just like constructions using straight edge and compass, constructions through paper folding is both mathematically interesting and aesthetic, particularly in origami. There are many other beautiful shapes that can be constructed through paper folding. Surprisingly, it turns out that origami is much more powerful than straight-edge and compass creations, because many things that cannot be created using straight-edge and compass, such as the doubling of a cube and trisection of an angle, can be created through paper folding. It's a great avenue in mathematics to learn and discuss about geometry.

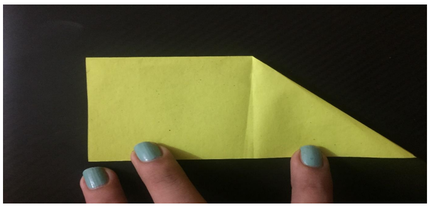
Let's make it:

Requirements: 8 square shaped pieces of same size as shown here in the figure.

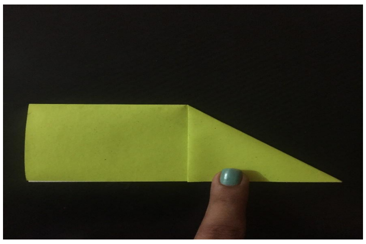




Step 1: Fold the square paper in half on the horizontal axis and crease well.

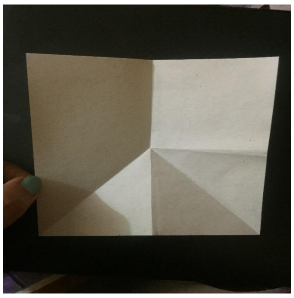


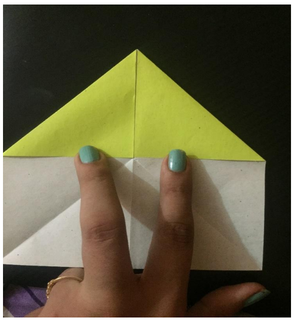
Step 2: Fold the right corner downwards such that it coincides with the center of the bottom horizontal line. Crease well.



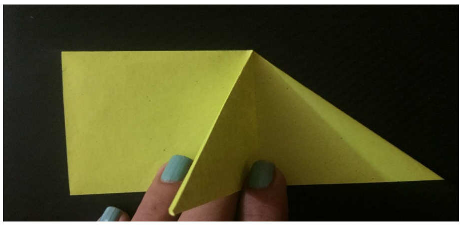
Step 3: Unfold the fold of step 2. Fold the triangle inside along the crease.

Step 4: Unfold the square. Fold the upper corners such that it meets at the center of the square.

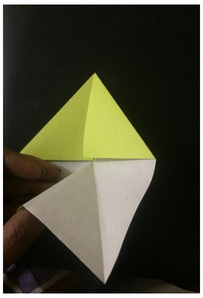


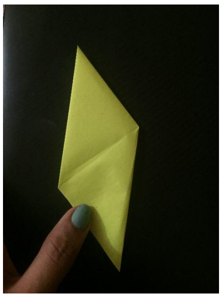


Step 5: After step 4. Make again the fold of step 3. The basic unit shape for the figure is ready.

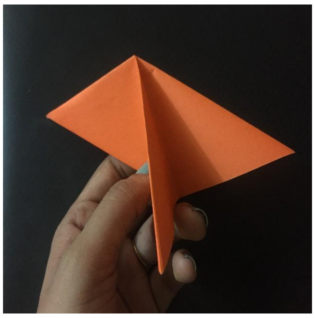


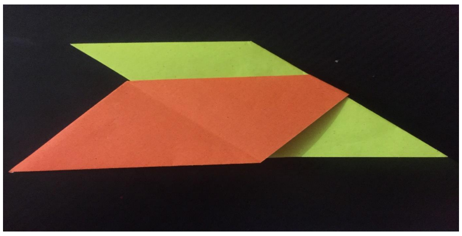
Step 6: Join the two units. Putting one parallelogram unit between the double flaps of another parallelogram unit. Fold the flap of the left unit to lock the right unit into the place.

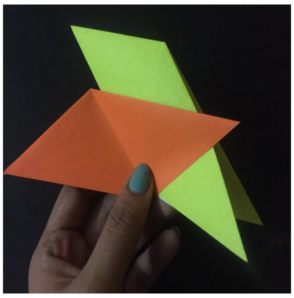




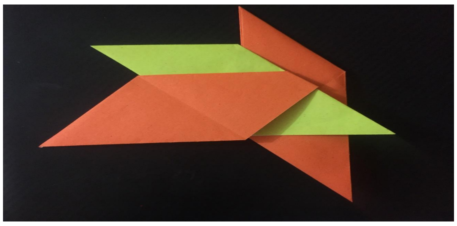
Step 7**:** Add another unit to the flap of the right one and also lock them into place.

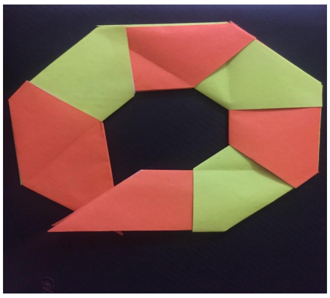






Step 8: Repeat the step 7. Keep adding and locking units until all 8 units are used.

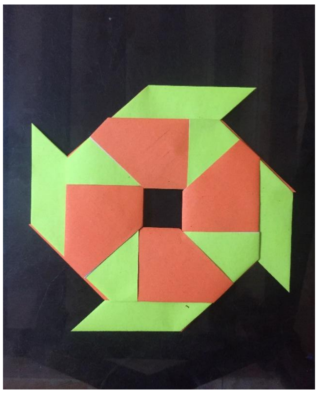


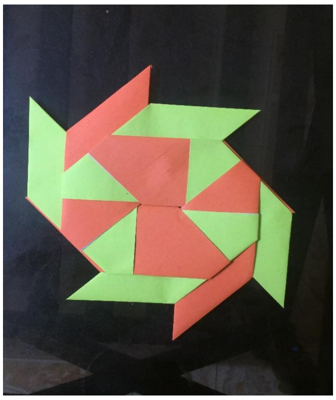


Step 9: Lock the last unit carefully to the first one.

Step 10: Transform the Frisbee into an 8-point ninja star.

For each step mentioned above, following are the supporting pictures.





Possible Mathematical investigations:

With students one way to work is to teach them how to make the 8-pointed star. Here we describe another way, where the students first observe the star and describe each other to figure out its construction.

- Ask students to observe the figure carefully. Encourage them to describe the figure in detail in their own words. The description should be such that the person hearing the description can visualize the figure in their mind. The students who are listening to the person describing will make attempt of drawing the figure described to them.
- Ask students about the size, dimension and properties of the formed shape.
- Ask students what they think will be the basic unit of this figure. Guess the original shape of the basic unit of the figure. How many basic units this figure consist.

- Ask students to state the necessary and sufficient properties of the original shape of basic unit. (Properties of square)
- Ask students to observe the figure formed. Ask the students to describe the shape formed in term of sides, corner, size, area, perimeter, and measure of angles.
- Concepts such as properties of trapezium, area of trapezium, area of square, area of rectangle, perimeter of square, perimeter of rectangle, perimeter of trapezium, triangle, and right—angled triangle could be discussed with the students.
- Ask students to observe how the 8-pointed start is formed. This we can call as a transformation in star making, where at each step one edge is added to make it a start shape. (With each transformation two points are adding in the figure. Within four transformation the figure is going to transform into 8 point ninja star)
- Encourage the students to try with their own hands the transformations

16-pointed Star

Objectives:

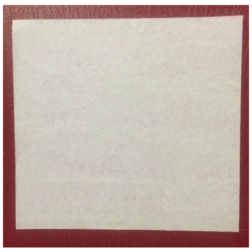
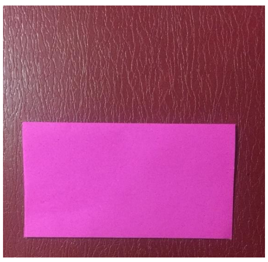
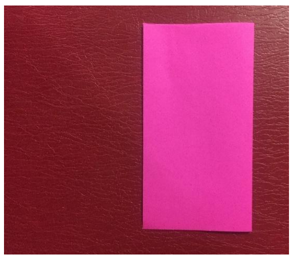
- To encourage the use of appropriate mathematical language and ways to describe a shape
- To provide an integrated approach to use understanding of different types of quadrilateral - square, rectangle and triangles- scalene, isosceles, right angled triangle.

Introduction:

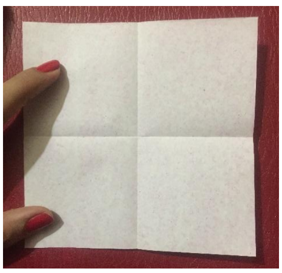
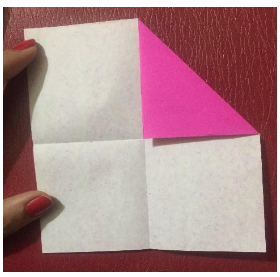
Generally geometry is of weakness among students. There are very few avenues to discuss geometry in classroom and origami is one of them. Origami activities are great tool to strengthen an understanding of geometric concepts, formulas and labels making them come alive.

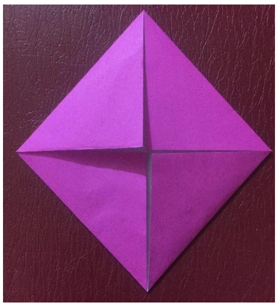
Let's make it:

Step 1: Start with the square sheet of paper. If it is not square make it square. Fold it horizontally half, crease it well and open fold again vertically half, crease well.

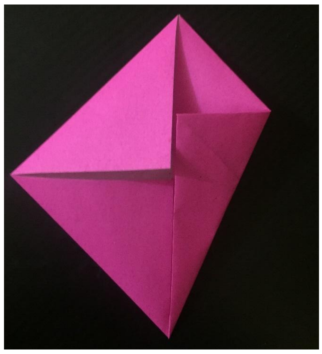
Step 2: Open the fold of step 1. Fold the corner of the square towards the center along the crease.

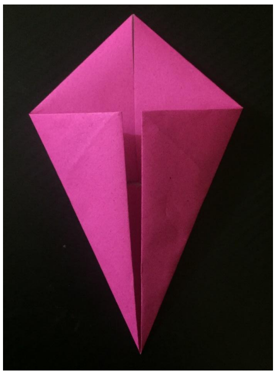





Step 3: Repeat the last step for all the corners. Fold all the 4 corners in to meet at the center.

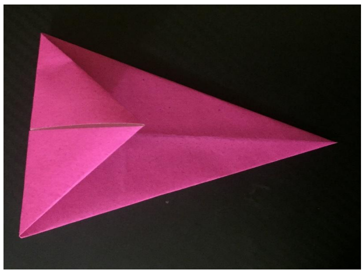
Step 4: From the one corner make a vertical fold inside to meet the center crease.



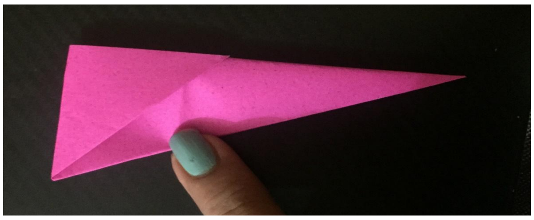


Step 5: Now, from the opposite corner make the similar fold like last one.

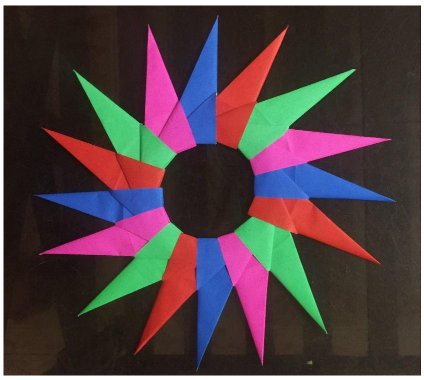
Step 6: make a fold backward as shown and crease well.



Step 7: make a fold from the center inward. This is one unit we need 15 more units. How can we join these units together without using glue? Each unit has ‘pockets’ and ‘tips’



Step 8: starting with two units, fit tips on unit into the pocket of another as shown in the picture. Repeat the step fitting the tips into the pocket attaching one unit to the other until we use up all our units.



Possible Mathematical investigations:

- Revisiting the concepts of quadrilaterals, triangles, area and perimeter, fraction.
- Ask students to differentiate between this star and the star they draw in general.
- Ask them to describe the star in detail.
- Ask students to draw the star by looking at it. Encourage the students to mention how the drawing of the star can be made more accurate in term of size, dimensions and shape.
- Talk to students about the color pattern of the star.
- Ask students to think of another pattern for the star, then to figure out how many sheets of which color they will need to create the pattern that they thought about.
- Ask students how to convert the rectangular sheet into square shaped piece. Also, which property of the square will be required to verify this conversion?
- State the differences between the properties of square and rectangle.
- Instruct the students to observe each fold very carefully. At each step observe the change in the shape. Try to distinguish the new shape formed from the previous one in terms of number of sides, area, perimeter, and angles and try to think of as many relations across them as they can.
- Ask students how much area and perimeter of the figure will change after the first fold. Say the size of the side of initial square was ‘a’.
- Can we say that the figure now formed is a square? How much is the area and perimeter of the formed square in relation to the initial square.
- Ask students to observe the fold of triangle. What they can say about the triangle formed.

Other relevant information /miscellaneous:

The art of origami or paper folding has received a considerable amount of mathematical study. Fields of interest include a given paper model's flat-foldability (whether the model can be flattened without damaging it) and the use of paper folds to solve mathematical equations. The Huzita–Hatori axioms are an important contribution to this field of study. These describe what can be constructed using a sequence of creases with at most two point or line alignments at once.

References:

http://www.langorigami.com/article/huzita-justin-axioms

3D-Paper Crane

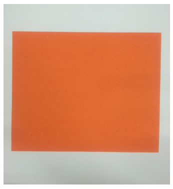
Objectives:

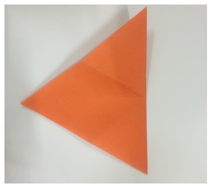
- To improve spatial visualization skills using hands-on learning
- To improve the problem solving abilities of students
- To improve 3D perception, logical thinking, focus and concentration
- To develop their understanding of symmetry.

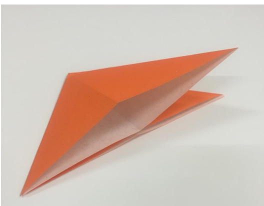
Introduction:

Transforming a flat piece of paper into a 3D crane is a unique exercise in spatial reasoning. This origami activity is also very important tool in teaching **symmetry**; for most of the folds whatever is done on one side is done on the other. It allows students to create and manipulate basic geometric shapes. As Piaget said "motor activity in the form of skilled movements is vital to the development of intuitive thought and the mental representation of space", such activities are critical in developing spatial understanding.

Let's make it:
Step 1: Start with a square sheet of paper. Next, fold the square in half on diagonal.

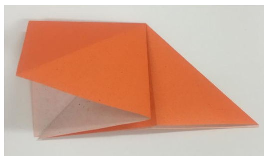


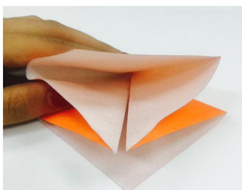




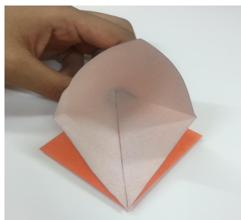
Step 2: Fold the paper in half from right to left, diagonally.

Step 3: Take the top flap and open it, creasing the left and right sides so you can fold the top/right corner to the bottom corner.

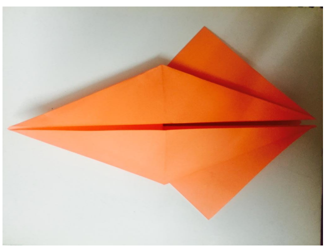


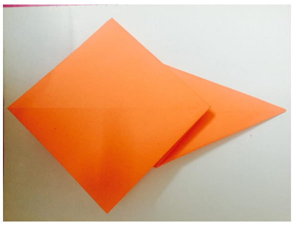


Step 4: Turn the paper over and do the same thing to the other side. To do that, lift the left flap up so it stands vertically like seen in the picture.

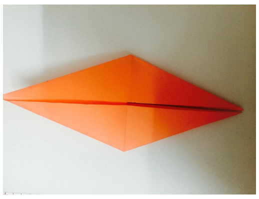


Step 5: Grab the left and right sides of the flap and open it up. Crease the sides so you can fold the top corner down to the bottom.



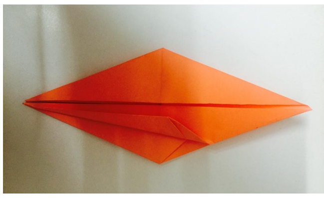


Step 6: Open the flap upwards. Fold the left and right sides inward. Crease it well.



Step 7: Flip it over and repeat step 5 & 6. It should look this when you are done.

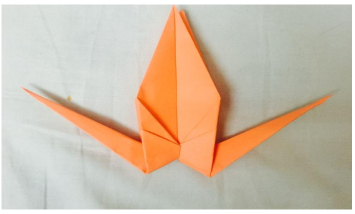
Step 8: Take the upper layer of both sides and fold the lower parts into the center like you see in the picture.



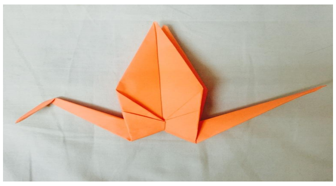
Step 9: Turn the paper over and do the same thing to the other side.

Step 10: Take the bottom flap and fold it over to the top like you see in the picture.

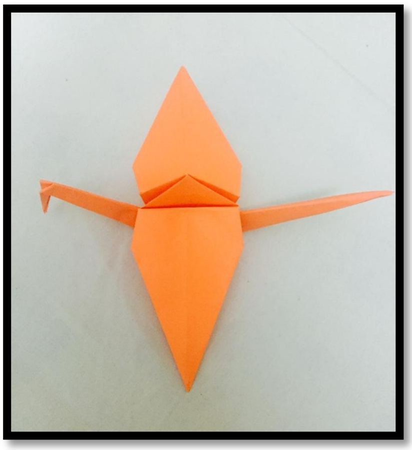
Step 11: Do the same thing to the other side. Along the crease formed in previous step insert the flap inside.



Step 12: Take one of those pieces that you pulled apart, and slightly open the top corner so that you can bend a portion of it down to form the head. After bending a portion down, crease the sides of the head up so the piece will stay bent.



Step 13: Bend the wings down at a 90-degree angle. And beautiful origami crane is ready.



Possible Mathematical investigations:

- Ask students to identify the shapes formed at different steps.
- Discuss the characteristics of the different shaped formed.
- Encourage students to describe mathematically how one fold is different or similar to another fold.
- Ask student to find the symmetry in every fold.
- To find out the axis of symmetry.
- Encourage students to predict the next fold in between different step.
- Encourage students to find out different ways to make crane.

Other relevant information /miscellaneous

Origami is the art of paper folding, which is often associated with Japanese culture. The goal is to transform a flat sheet square of paper into a finished sculpture through folding and sculpting techniques. Origami practitioners generally discourage the use of cuts, glue, or markings on the paper. The small number of basic origami folds can be combined in a variety of ways to make intricate designs. The best-known origami model is the Japanese paper crane.

References:

Lang, Robert J. *Origami Design Secrets: Mathematical Methods for an Ancient Art.* A. K. Peters, Ltd., 2003.

Lang, Robert J. *The Complete Book of Origami: Step-by-Step Instructions in Over 1000 Diagrams*. Mineola, NY: Dover Publications, 1988.

Fun with Triangles

Objectives:

- To recognize and explore the properties of triangles
- To identify and examine symmetry in geometry in geometric figures
- To make use of/apply the properties of triangles

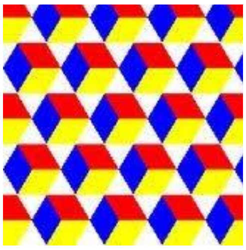
Introduction:

Many middle school students find it difficult to master the concept of triangle. This difficulty arises because of the focus on the memorization of the different types and properties of the triangles without proper understanding. This activity gives the opportunity to analyze the differences between different types of triangles and making the use of properties of triangles. Through this activity an attempt has been made to relate mathematics to everyday life. Tessellations provides various possibility with more than one way of forming the same shape encourage student's creativity. A tessellation is a regular pattern made up of flat shapes repeated and joined together without any gaps or overlaps. These shapes do not all need to be the same, but the pattern should repeat. Another word for tessellation is tiling.

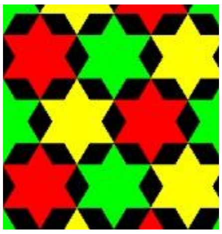
Regular tessellation is a highly symmetric tessellation made up of congruent regular polygons. A regular polygon is one with all sides equal and all it's interior angles equal. There are only three regular polygons that can be used to make regular tessellation, the triangle, square and hexagon.

The reason for this is the angle measures of these polygons. The angle around each vertex in a tessellation has to be 360 degrees. Therefore the interior angle degree of a regular polygon should divide into 360 evenly. The interior angles of the triangle, square and hexagon are 60, 90, and 120 respectively which works. There does not exist any other regular polygon with this property.

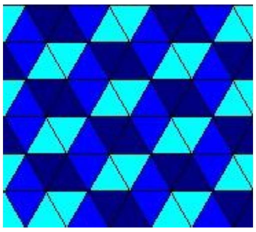
Possible Mathematical investigations:



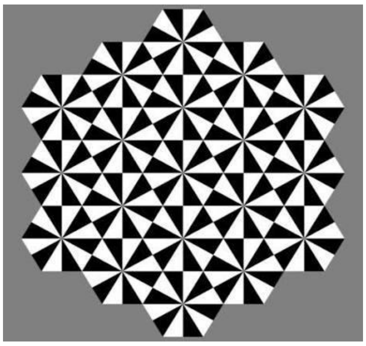
- Discuss with students about the properties tessellations such as vertex condition.



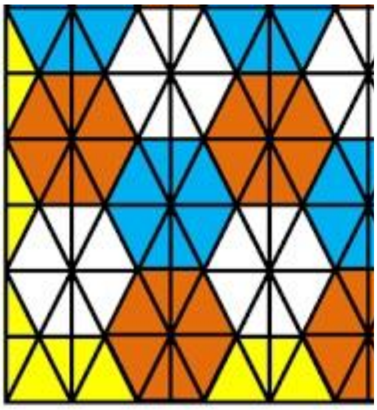
- Give some time to students to play with triangles and form different Shapes- Square, rectangle, parallelogram.
- Ask students about what are the types of triangles, which will give square, which one will not give square. Encourage the students to think and give explanation.



- Ask the student about what are the different possibilities to form hexagon with given set of triangle.
- What are the specific types of triangle and what is unique in their properties that will give the hexagon by minimum number of triangle?



- Ask students about the relationship between area and perimeter with the change in the basic tessellation by giving some algebraic dimension to triangle. Give them real life word problems where tiles are getting use.



- To find out how many tiles of a particular shape will be required to cover a specific area.
- Showing the various pictures of triangle tessellation. Encourage students to talk about the similarity and differences. Ask students to make/draw their own tessellation. This activity can be extended to the construction of different types of triangles to make your own tessellation kit.

Other relevant information /miscellaneous

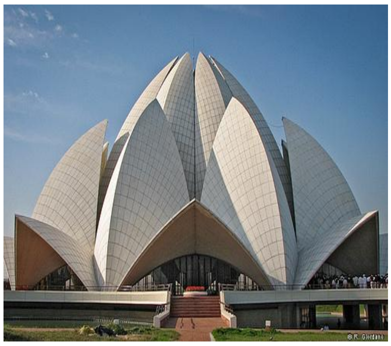
Tessellation is a system of shapes, which are fitted together to cover a plane, without any gaps or overlapping. The word tessellation itself derives from the Greek tesserae, which is associated with four, square and tile. Presumably this is an indication of the fact that tiles of this shape are the easiest to interlock. Tessellations are a common feature of decorative art and occur in the natural world all around us. Two people have principally been responsible for investigating and developing tessellations: Roger Penrose, an eminent mathematician, and the artist, M.C. Escher.

Symmetry in Nature

Objectives:

- To develop the understanding of symmetry that surrounds us in our daily life.

Introduction:



Symmetry is an important geometrical concept, commonly exhibited in nature and is used almost in every field of activity. Artists, professionals, designers of clothing or jewelry, car manufacturers, architects and many others make use of the idea of symmetry.

A geometric shape or object is symmetric if it can be divided into two or more identical pieces that are arranged in an organized fashion. This means that an object is symmetric if there is a transformation that moves individual pieces of the object but doesn't change the overall shape. The type of symmetry is determined by the way the pieces are organized, or by the type of transformation:

- An object has reflection symmetry (line or mirror symmetry) if there is a line going through it, which divides it into two pieces, which are mirror images of each other. https://en.wikipedia.org/wiki/Symmetry - cite_note-7
- An object has rotational symmetry if the object can be rotated about a fixed point without changing the overall shape.
- An object has translational symmetry if it can be translated without changing its overall shape. https://en.wikipedia.org/wiki/Symmetry - cite_note-9
- An object has helical symmetry if it can be simultaneously translated and rotated in three-dimensional space along a line known as a screw axis.

- An object has scale symmetry if it does not change shape when it is expanded or contracted. Fractals also exhibit a form of scale symmetry, where small portions of the fractal are similar in shape to large portions.

Let's explore it-

The beehives, the flowers, the tree-leaves, religious symbols, rugs, and handkerchiefs — everywhere you find symmetrical designs.

Possible Mathematical investigations

- Ask the students to list symmetric things that they commonly find in their daily life.
- Like utensils, hardware, stationary objects, objects of the classroom, buildings, images of different monuments, vegetables and fruits, watch, animal bodies, clothes etc.

- Check which of these are symmetrical or asymmetrical.
- Ask them to categories these in different kind of symmetry.
- Encourage them to identify the lines of symmetry for those things, which are symmetric.
- Discuss with the students about the differences in symmetry of different figures.
- Can we find the axis of symmetry in irregular figures? Think about it.

Other relevant information /miscellaneous

Other symmetries include glide reflection symmetry and roto-reflection symmetries.

References:

http://www.math.ucla.edu/~radko/circles/lib/data/Handout-817-915.pdf

Be my mirror

Objectives:

- To enhance the understanding of reflection symmetry in daily routines

Introduction:

Reflection symmetry, line symmetry, mirror symmetry, mirror-image symmetry, is symmetry with respect to reflection. That is, a figure, which does not change upon undergoing a reflection, has reflectional symmetry. In 2D there is a line/axis of symmetry, in 3D a plane of symmetry. An object or figure, which is indistinguishable from its transformed image, is called mirror symmetric. Mirror is something the every individual look at while going out of the house. Mirrors have become an important part of life of every individual. Student in middle grades have learned the word reflection but they generally face difficulty in developing the understanding of its characteristics. And to develop their understanding the games are an important tool. Games provide opportunities for students to practice important mathematical concepts and skills and to develop and deepen their mathematical understanding and reasoning.

Let's play the Game:

This game is to be played in pair. In which one will be himself and other partner have to be his mirror by standing in front of him. The individual who is acting like the mirror have to reflect the action the other person is doing. Initially the action of the individual will be random but seeing that the mirror is easily giving the reflection.

After a while the individual will start making conscious efforts to perform action that the individual playing the role of mirror will find difficult instantly. Later at the advance level the game can be made complicated by four people playing the role of four mirrors in the four directions around that person.

Possible Mathematical investigations:

- Discuss with students what were the important things to be kept in mind while playing this game.
- Which were the action the person playing the role of mirror was finding difficult to reflect instantly.
- Ask student where will be the locus of symmetry. How to determine the locus of symmetry.
- While playing the advance level how the things changed from the previous one.

Thread painting.

Objectives:

- To enhance the understanding of reflection symmetry in paintings and design

Introduction:

Thread painting is an art that could be done in simple ways as well as using complex methods. It is really a fun idea that children can be engaged with it. Here we introduce you to a simple and basic method of thread painting. We suggest few activities that could be done to connect thread painting with mathematics.

Let's make it:

Material Required: Thread, poster color, drawing paper

- Take a blank card or make a blank card yourself by folding a blank paper half.
- Take a long piece of thread. (not very long).
- Put the thread in paint. You can paint the thread in any color. Even you can paint it with two or more colors of your choice.

- Put the colored thread on the paper between the drawing sheet in any design you like. Leave one end of the thread coming out of the folded paper.
- Press the paper with your hand. And pull the string that you have not put inside the card.

- Further improvement: You may paint the thread in different color combinations along the length to achieve more exquisite designs

Possible Mathematical investigations:

- Ask student to observe the painting carefully. Ask them whether they are able to find any symmetry in the paper.
- What kind of symmetry?
- Where is the line of symmetry in the paper?
- How is this different from tracing a design? Does tracing also give reflection symmetry?

Kirigami

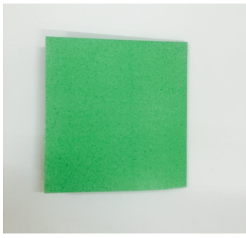
Objectives:

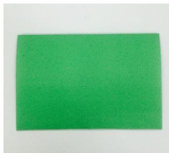
- To improve spatial visualization skills using hands-on learning
- To improve the problem solving abilities of students
- To improve 3D perception, logical thinking, focus and concentration
- To develop their understanding of symmetry.

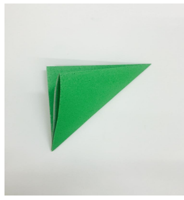
Introduction:

Kirigami is similar to origami in that it is a form of paper art. The major difference is that in origami, you fold paper whereas in kirigami, you fold and also *cut* the paper.

Let's make it:



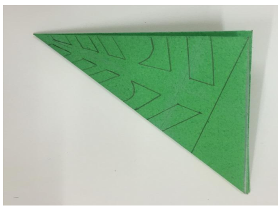




Step 1:Take a square sheet of paper.

Step 2:Make a horizontal fold.

Step 3:Next a vertical fold.

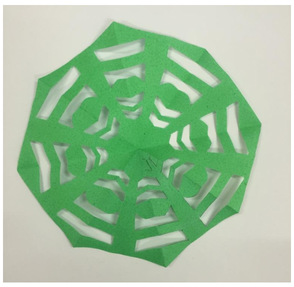


Step 4:Again fold the square diagonally.

Step 5: Draw your own design.

Step 6: Cut the design.

Step 7: Open all the folds. Your design is ready.



Possible Mathematical investigations:

- Emphasize on the relationship between the number of folds that you make and the line of symmetry.
- Which type of symmetry is there in this design?

- Where do we find such designs around us in our daily lives?

Other relevant information /miscellaneous:

In the United States, the term "kirigami" was coined by Florence Temko. She used the word kirigami in the title of her book, *Kirigami, the Creative Art of Papercutting*, 1962. The book was so successful that the word kirigami was accepted as the name for the art of paper cutting. In Japan, the word kirigami had been in use for a long time because "kiru" means to cut, and "gami" means paper. So, kirigami meant "to cut a paper".

Trace the shortest route

Objectives:

- To enhance spatial understanding and measurement skills
- To enhance the ability of conversions of units

Introduction:

Spatial thinking allow students to comprehend and analyze phenomena related to the places and spaces around them—and at scales from what they can touch and see in a room or their neighborhood to a world map or globe. Spatial thinking is one of the most important skills that students can develop as they engage with maps. It also deepens and gives a more complete understanding of history and is linked to success in math and science. Young students also enhance their language skills as they collaborate and communicate about spatial relationships. Students who develop robust spatial thinking skills will be at an advantage in our increasingly global and technological society.

About Scale of a Map:

As a map has to represent a portion of earth's surface accurately, each map has to have a "scale" which indicates the relation between the distance on the map and the actual distance on the land. The map scale is typically shown in the legend box of a map, along with other symbols that provides useful information about the map.

Trace the shortest route: During this activity basically students are required to find the different routes to reach from one location to another location using map. Then they have to find the distance of all routes using scale of map. Finally telling which one is the shortest route between two location.

Possible Mathematical investigations:

- Developing spatial thinking in finding out different routes for reaching one place to another.
- Enhancing measurement skill.

- Conversion of units
- Understanding the use of scaling.
- Exercise for reading and familiarize with Sign & symbols.

Other relevant information /miscellaneous:

A ratio or representative fraction specifies how many units on land is equal tone unit on the map. For example, a map showing a scale of 1/ 100000 or 1:100000, tells us that one centimeter on the map is equal to 100000 centimeters i.e. 1 kilometer on the Land. The ratio is always mentioned in the map, such as "one centimeter equals one kilometer" or "one inch equals ten miles".

A graphic scale also known as the bar scale is a line that specifies the distance in kilometers or miles as they show on a map; even as the map is enlarged or reduced in size the line has an advantage of remaining accurate. Maps can be recognized as large scale or small scale maps. A large-scale map shows much more detail than a small-scale map.

Room Interior

Objectives:

- To enhance spatial understanding and measurement skills
- To enhance the skill of scaling
- To enhance the ability of conversions of units

Introduction:

We human beings often struggle for deciding the interior of room/house. Whenever we buy new thing for room/house we struggle where to keep that thing. We are not able visualize how the things will look and we start manually moving the furniture that increase lots of work. Thus, the activities like this where students work with cutouts of furniture and try to fit as many way possible and come out with best possible arrangement. All this with scaled down

Let's make it:

Scale drawing of the room on squared paper would be useful.
Pieces of card can be cut to represent furniture (scaled down) and moved around to obtain suitable circulation space.
In planning the carpet consideration needs to be given widths available.

Possible Mathematical investigations:

- Working with shapes to make scaled- down cut outs of furniture
- Conversion of units to scale down
- Enhance problem solving and develop visualization skills.

Creating Home

Objectives:

- To enhance spatial understanding and measurement skills
- To enhance the skill of scaling
- To enhance the ability of conversions of units

Introduction:

Let's make it:

- Make sketches of homes of friends and relatives with their floor plans.
- Look at an architect's drawing of a house.
- Make a list of the rooms you want to include in your design.
- Make cutouts of the floor plans of the rooms you want to include to a suitable scale and then investigate ways in which they might fit together like a jigsaw.
- Keep in mind to connect the rooms conveniently, to keep provision for a bathroom or toilet.
- Draw elevations to make your design clear.
- Make an isometric or perspective drawing of your design.

Possible Mathematical investigations:

- Conversion of units to scale down
- Enhance problem solving and develop visualization skills.

Reference:

http://www.sutd.edu.sg/cmsresource/faculty/saikit/projects/furniture/ucla_math.pdf

Clock Mathematics

Objectives:

- To develop the understanding of angles using clock

Introduction:

Clock angle problems relate two different measurements: angles and time. The angle is typically measured in degrees from the mark of number 12 clockwise. The time is usually based on 12-hour-clock.

A method to solve such problems is to consider the rate of change of the angle in degrees per minute. The hour hand of a normal 12-hour analogue clock turns 360° in 12 hours (720 minutes) or 0.5° per minute. The minute hand rotates through 360° in 60 minutes or 6° per minute.

Possible Mathematical investigations:

- Find out in how much time minute hand will cover 1°.
- Find the angle measure between the hour and minute hands of the clock in each figure. 9:00, 1:00, 6:00 , 2:20