Analyse Interpret Decide (AID) discussion of Scenarios

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Scenario 1

In Ragini madam's class students were asked to fill in the appropriate sign < or > or =, in the following comparison of expressions

$$59 \div 42 \quad \square \quad 359 \div 342$$

Shamin said she would write the equal sign because

> " 59 ÷ 42 = 1 Remainder 17
>
> 359 ÷ 342 = 1 Remainder 17
>
> So in both the expressions the answer is 1 and the remainder is 17, and that is why they are equal."

How would you respond to Shamin?

Scenario 2

Ms. Shabana's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem.

Student a:
- Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

Student b:
- Check to see whether 371 is divisible by any prime number less than 20.

Student c:
- Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Student d:
- Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

How would you respond to each of the student?

What could be the next course of actions in this class?

Scenario 3

Mr. Ajay was reviewing his students' notebooks. While doing so he noticed that one of the students — Gunjana has invented an algorithm that was different from the one taught in the class. Gunjana's work on one of the problems looked like this:

$$\begin{array}{r} 983 \\ \underline{\times \quad 6} \\ 488 \\ \underline{+\ 5410} \\ \underline{5898} \end{array}$$

What is Gunjana doing? Is she right?

Can you use her method to calculate 678×9?

How would you respond to her?

Scenario 4

Mr. Rodrigues taught his students that a number is called "abundant" if the sum of its proper factors exceeds the number.

For example, 12 is abundant because $1 + 2 + 3 + 4 + 6 > 12$.

He asked students to figure out other numbers that are abundant.

On a homework assignment, many students incorrectly recorded that the numbers 9 and 25 were abundant.

Mr. Rodrigues is thinking about the ways that might have led to such answers. Help him by listing out all possible reasoning that students might have done in this exercise.

And figure out how can he respond to these students.

Scenario 5

Leena madam just finished teaching equivalence of expression in her class. She asked her students to explain why the expression $a-(b+c)$ and $a-b-c$ are equivalent.

Here are some responses:

Charu: They're the same because we know that $a-(b+c)$ doesn't equal $a-b+c$, so it must equal $a-b-c$.

Heena: They're equal because of the associative property. We know that $a-(b+c)$ equals $(a-b)-c$ which equals $a-b-c$.

Gurinder: They're equivalent because if you substitute in numbers, like $a=10$, $b=2$, and $c=5$, then you get 3 for both expressions.

Tabu: They're the same because of the distributive property. Multiplying $(b+c)$ by -1 produces $-b-c$.

How would you respond to each student, such that their mathematical understanding is enriched?

Scenario 6:

> My intentions were to allow students to discover the rules for themselves. The way I did that was to develop a pattern, so I started with $2^5, 2^4, 2^3, \ldots$ and they were supposed to discover a pattern ... students were to see a pattern and to use that pattern to come up with the rules for the zero and negative exponents.
>
> A typical chart for such an exercise might look something like the one below along with expected pupil responses:
>
> | **Power** | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 | 2^{-1} | 2^{-2} | 2^{-3} | 2^{-4} |
> |---|---|---|---|---|---|---|---|---|---|
> | **Expected Pupil Response** | 16 | 8 | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |
>
> To her surprise, however, Julie found that the patterns the students discovered were not exactly what she had in mind.
>
> First of all, students had difficulty discovering the pattern to start with. They wanted to say that numbers were decreasing by 2 instead of being divided by 2 as they went down, and so they came up with 2^0 is equal to zero, 2^{-1} is -2, and 2^{-2} is -4, and I'm not sure whether that's because that's the pattern they were seeing or just because people see 2^0 and they want to say it's 0 and 2^{-1} - multiply them together so they get -2. I'm not sure but that's what they came up with.

How can you convince these students mathematically that they are wrong?

Scenario 7:

Majority students in Mrs. Ragini's class said that 3/4 is same as 5/6. They said they are equivalent fractions. For that the students gave the following reasoning.

- In both cases there is one less than the denominator, and therefore if we shade it, one piece would be left
- If you add 2 to 3 you get 5, and if you add 2 to 4 you get 6. There is a common factor here, and therefore they are equivalent.
- If we make the denominators common that would be 6, and numerators too become the same.

How would you respond to these students?