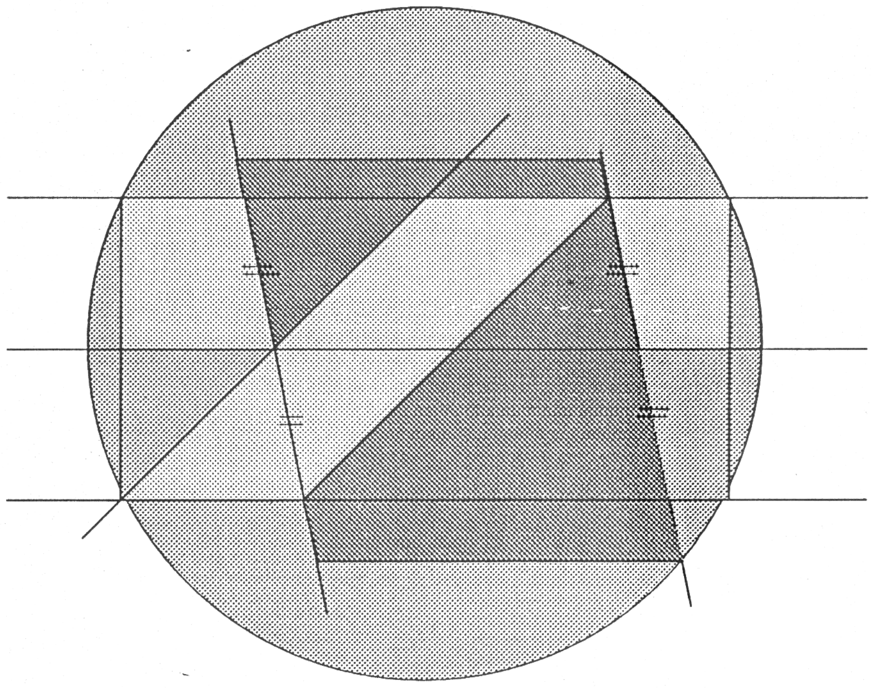
Geometry through Collage

Boosting Performance in High School Geometry



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Contents

Introduction

High-school mathematics teachers are often in a dilemma about which category of students in their class they must mainly address. If the teacher aims his instruction at the students who perform very well, she or he is able to obtain a quick response. This brings in turn immediate satisfaction to the teacher. Addressing the needs of under achievers and improving their performance consumes much time and effort. There is always the possibility that all the effort and time spent may still not produce results. There is also the danger of losing the attention of the other students in the class. It comes as no surprise then that a teacher decides to aim his instruction at the 'average' student in his or her class. In the process the students who perform poorly are usually left behind, even though they are the students who most need a teacher's help.

Advances in technology have made a number of new resources available in the mathematics classroom. Not all of these however have led to a definite improvement in classroom learning. The OHP and the slide projector are attractive gadgets to use, but in the hands of an enthusiastic teacher they are often used to increase the volume of material transferred to the student or the rate of transfer. The under achievers turn more passive in such a situation. Calculators and computers are certainly very useful and are used with much benefit by students who perform well. Most under achievers however conclude that it is not suitable for their learning abilities. In our present social context, where computers are still accessible only to a few students, under achievers tend to shy away from using them.

However the situation is not hopeless. In this book we describe a method of using collage art work in geometry teaching. Collage art-work has the advantage of being very simple and also of already being done in many schools as part of craft activities. One of the authors used it in his classroom and found that it was remarkably effective. It showed us that under-achieving students are capable of learning even complex reasoning patterns if they are sufficiently motivated. It also showed that under achievers do not lack intelligence and that in fact, they show a lot of intelligence in the right situations. Another pleasant surprise that emerged in implementing the method was that even students who perform well in geometry enjoyed the method and benefited considerably in terms of learning and understanding geometry.

The under-achieving student

It is not quite true that under achievers try hard to learn but fail to do so. In fact, in the classroom, they use every trick they can think of to avoid learning — day-dreaming, hedging, giving incomplete answers, looking for facial and voice prompts from the teacher and from friends, using the join-in strategy (joining voice with another student when he or she is answering a question), excessive questioning and making excuses are some of the milder variety; disrupting the class, refusing to work, staying away from class, annoying other students are not so mild. These tactics are highly provocative, and it is understandable enough that most teachers would conclude that "the students don't care anyway, so why should I put in extra effort to teach them mathematics?" But this eventually amounts to shirking professional obligation. Teachers must seek to create an environment where atleast an expected minimum of mathematical literacy and quantitative skills is gained by all students. It is true that for most of the students who perform poorly at school the home environment is an important factor. However our teaching needs to offset these adverse factors which cannot be controlled, rather than add to them.

One of the authors, Bhatt, concerned about the poor performance of a section of students in his class, was struck by something that he noticed one day when he visited his class during the craft period. The so-called 'poor performers' in geometry were very actively involved in the art class, cutting paper, making patterns and so on. This started him on a train of thought about how to link geometry teaching to craft work. The method that eventually evolved of using 'collage art work' to teach geometry is presented here. The author was quite impressed with the improvement in the performance of most under achievers at the end of the year when collage work was used to teach geometry.

Collage art method

Collage is a form of art in which objects, often bits of pictures or coloured paper, are glued to a backing to form a picture. In order to teach geometry, we create a collage of a geometric object which is associated with a theorem or a problem. Each student creates a collage picture of the problem and this is used as a base for answering questions on an activity sheet. The activity sheet is designed to help students discover the patterns in their picture and to reason about the geometric properties involved. Some selected samples of the collage pictures and associated activity sheets are presented in this book.

There are a few points to be noted about the collage art method. The first concerns time available for instruction. It might appear at first that the time available is insufficient and hence that the collage method is not feasible in practice. But there are ways of getting round this. It is only the initial few sessions which take time, since the activity is new and the students are getting used to the demands of accuracy required for making good geometric figures. The explanation and work on the activity sheet also takes more time initially. However, when once the method has settled, things tend to proceed smoothly. Help from the art teacher is an added advantage, since some of the pictures can be constructed in the art class. Further, as the classes proceed, the cutting and pasting of figures can be given as home-work, something that the students often love to do. The enthusiasm for the activities could also be so high that it might be possible even to arrange extra sessions on Sundays, as happened in the author's class. The author was able to do 36 different problems in about 120 sessions. The topics covered were angle relations, congruent figures, parallelograms, area and linear symmetry.

One-sided glazed paper is best for collage work in geometry. Four to five different colours are sufficient for all the problems. All the students individually make the collage figure for each problem discussed in the class. With about five sheets of each colour (25 sheets in all) nearly 30 to 35 problems can be done. A drawing book is convenient to provide the backing paper for pasting the collage. The teacher can give step by step instructions in the classroom so that all the children go through the steps of cutting and pasting together. Cutting is best done with a paper cutter using a ruler. The paper cutter is also less expensive than a pair of scissors. Once the collage for a particular problem is ready, the worksheets can be distributed, discussed and filled up. The activity sheets need to be filed separately and the main conclusions or theorems for each activity can be copied separately into a notebook. After a few collage figures are made, the students can be asked to make the collage at home and the worksheets can be done in the class.

Precautions to be observed

The pieces of coloured paper that are glued to makeup the figure need to be cut accurately for the complete figure to turn out right. It is better to insist on such accuracy right from the beginning. Another important thing is to have the activity sheet ready and to get all students to complete the sheets. This is very important in order to meet the instructional objectives. So care must be taken in preparing the work-sheets and in helping the students to read and answer them. Initiating group discussion as the students work on the sheets is also essential and helps the students to clarify and express their ideas.

The major steps for the collage work sessions are:

1. Selection of the theorem or problem to be solved: The problem must be important from the instructional viewpoint. Most problems presented in the textbook are amenable to being taught by the collage method.

2. Preparing a sketch of the solution to the problem: The figure plays the central role in this method. Hence the construction of the figure, the number of different objects to be glued and their shape, size and colour must be clearly explained to the students.

3. Cut the different shapes accurately and glue them on to the backing.

4. Answering the worksheet: This requires a slow working through the solution while identifying the necessary part of the figure during explanation. The students must be encouraged to answer the questions themselves through mutual discussion and help from the teacher.

5. Finally copy down the conclusions mentioned in the worksheet into the notebook.

How the collage method works

Why does collage activity hold promise for teaching geometry to poorly performing students? A reason that immediately strikes one is that with the collage method the students are learning by doing. They are engaged in concrete activity, cutting and handling shapes. Concrete activities such as cutting and pasting to form the required geometrical shapes call for considerable thinking and planning and for problem-solving in a concrete sense. This forms a good base for more abstract thinking.

Another reason is that the aesthetic element and the element of skill involved is very attractive to this group of students. There is a great deal of motivation induced by the nature of the activity itself and in the hands of a friendly teacher this motivation could be transferred to learning geometry. Thirdly, the shapes that are constructed out of coloured paper have a 'solid' or 'filled' quality to them compared to line drawings which makes it easier to focus on sub-elements of the figure. Fourthly, there is a clear sense of completion of a task at the end of a topic. The student is able to show an object that he or she has created and earn praise for it. Not only do the students get an opportunity to do something, but also to talk about it in and out of class. The talk about the colorful product made by the student must be turned skilfully by the teacher to talk about the geometric aspects of the figure. Students must be encouraged to explain the geometry involved in the figure that they have constructed, the meaning of the various symbols and so on.

The manner in which a topic or a problem is covered in the work sheets is not strictly deductive, although a rough deduction of the solution is followed in moving through the steps. Questions are posed in each step with the objective of helping the student focus on a relevant portion of the figure and recall some associated geometric fact, or in order to discover a pattern in the figure. The approach is certainly not inductive. The collage work aids in fixing important ideas in geometry in the mind of a student and in providing an occasion for reasoning about geometry problems.

Activity 1: Properties of parallel lines

Before doing this activity the students should know how to measure angles and length of the line segments.

Collage work:

Cut two identical triangles in different colours, say red and yellow ($\triangle ABC$ and $\triangle ADC$ in Fig. 1). Ensure that the triangles are scalene triangles and do not have a right angle. This is necessary in all the activities (excepting those where triangles with special properties are required) to preserve a sense of the generality of the properties we are studying.

Glue the triangles on backing paper with one of the triangles inverted so that they have one side in common (Fig. 1). Draw lines p and q at the base of the triangles. Draw the line r by extending the side common to the two triangles.

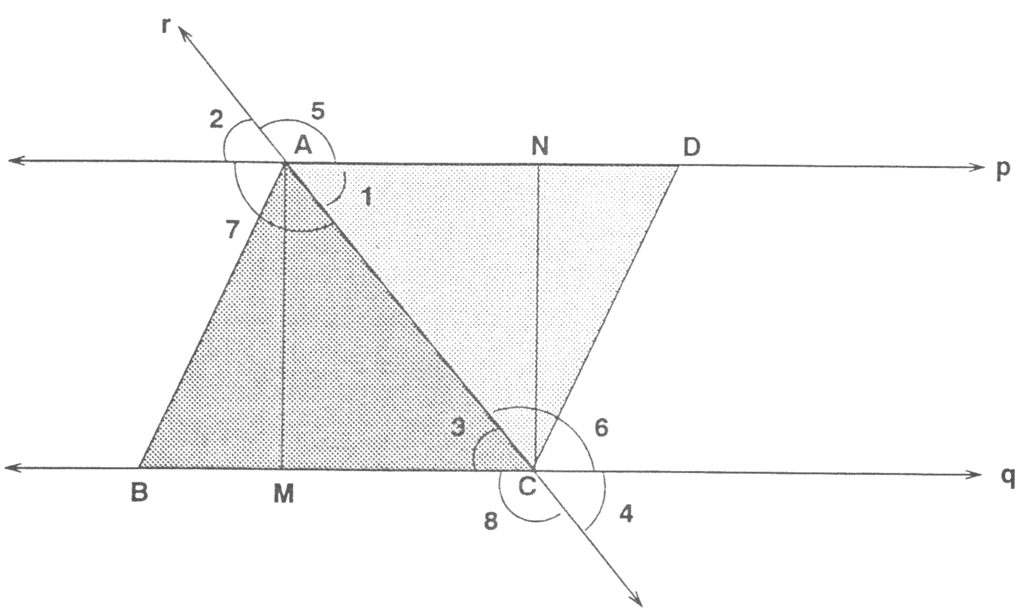

Fig. 1

Worksheet

1. Observe that there are two sets of parallel lines in the figure. Can you say which they are? _____||_____and _____||_____.

2. Measure the lengths AB and CD. AB = __________, CD = __________. Do you expect them to be equal? _____Why?

3. Draw the altitudes AM and CN of the two triangles. Length AM = __________, CN = __________. Hence _____ = _____.

4. We notice that AM and CN are equal just as AB and CD are equal. What is the property that is common to these pairs of lines?

   .____

   Ans: In each pair the lines are parallel and they lie between parallel lines p and q.

5. *Hence* ____________________*between* __________*lines are equal.*

   Ans: parallel intercepts . . . parallel.

6. Take AB and CD as parallel lines. Can you find the parallel intercepts between these lines which are equal? __________

7. What is r called in the figure in relation to lines p and q?

   _______________.

8. Measure the angles in the figure. $\angle 1$=_____, $\angle 2$=_____, $\angle 3$=_____, $\angle 4$=_____, $\angle 5$=_____, $\angle 6$=_____, $\angle 7$=_____, $\angle 8$=_____

9. What can you say about the measures of $\angle 1$ and $\angle 2$? ______
Why are they equal? ______________________________
Are there other pairs of equal
angles like this pair? ______________________________
Hence when two lines intersect, ____________________*angles are equal.*

10. Is $\angle 5 = \angle 6$? ______? They are equal because ______.
Can you find other pairs of angles which are similar to this pair? ______________________.
Hence when parallel lines are cut by a transversal,
______________*angles are equal.*

11. Is $\angle 1 = \angle 3$? ______________
What do you call such a pair of angles? ______________
Can you find another such pair of angles? ______________
Hence when parallel lines are cut by a transversal,
______________*angles are equal.*

12. $\angle 2 + \angle 5 =$ __________. The sum of these angles is 180°
because ______________________________.
$\angle 3 + \angle 6 =$ __________because ______________________.
Hence the sum of a ____________________*equals 180°.*
Can you name some other linear pairs of angles? __________

13. $\angle 1 + \angle 6 =$ __________. $\angle 7 + \angle 3 =$ __________.
Hence the sum when parallel lines are cut by a transversal, of ____________________*angles is equal to* __________.

14. In the figure, given that p, q are parallel lines and r is the transversal how many pairs of corresponding angles are present?
____

how many pairs of alternate angles are present? ___________
how many pairs of consecutive interior angles? ___________

15. *Hence when two parallel lines are intersected by a transversal the ________________ angles are equal, the ________________ angles are equal and the sum of ________________________ angles equals ___________.*

Activity 2: Angle sum property of a triangle

Before doing this activity the students should know that when two parallel lines are intersected by a transversal the pairs of alternate angles formed are always equal (see activity 1). They should also know about adjacent angles and the property of a linear pair of angles.

Collage work:

Cut a rectangular piece of coloured paper in say yellow, so that the length of the rectangle is about 5 cm. Now cut a scalene triangle (preferably not right angled) in a contrasting colour, say red, such that the base of the triangle is equal to the length of the rectangle, and the height of the triangle is equal to the breadth of the rectangle. Glue the triangle on the rectangle such that the base of the triangle coincides with the base of the rectangle (BC in Fig. 2) and the vertex (A) of the triangle lies on the opposite side of the rectangle. Number the angles as in Fig. 2.

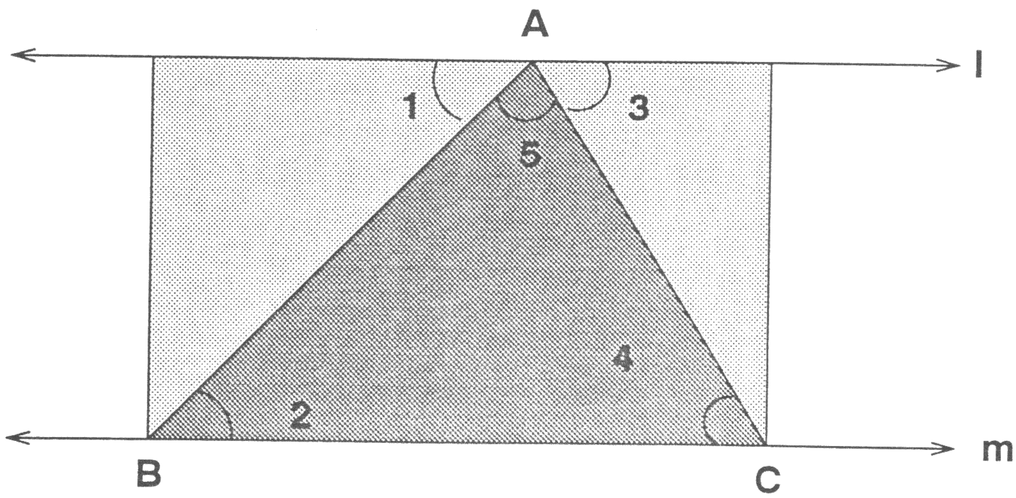

Fig. 2

Worksheet

1. Measure the angles in the figure. $\angle 1=$_____, $\angle 2=$_____, $\angle 3=$_____, $\angle 4=$_____, $\angle 5=$_____.
2. What can you say about the measures of $\angle 1$ and $\angle 2$? __________. Why would you expect them to be equal? ________________
3. Is $\angle 3 = \angle 4$? _____Why do you expect them to be equal? __
4. What do you expect to be the sum of $\angle 1 + \angle 5 + \angle 3$? __________.
5. Why must the sum of the above angles be equal to 180°? __
6. Now what can you say $\angle 2 + \angle 5 + \angle 4 =$ __________. Why it is so ? ______________________________
7. In the figure what are $\angle 2$, $\angle 5$ and $\angle 4$? ________________
   Ans: They are the three angles of the given triangle.

8. *Hence the ____________of three angles of a triangle is always equal to ____________.*

Activity 3: The exterior angle property of a triangle

Prior to doing this activity the students should know that the sum of all three angles of a triangle is 180°. They should also know the property of parallel lines.

Collage work:

Cut three identical triangles in different colours. Ensure that the triangles are scalene triangles without a right angle. Glue two of the triangles (△ABC and △ADC in the Fig. 3) on a white backing so that one of the triangles is inverted. The third triangle is needed only for the purpose of illustrating the external angle. Hence we retain only a portion of it (△DCE in the Fig. 3) and cut off the rest of the triangle by cutting along the altitude DE. While pasting the triangles on the backing paper, point out to the students that the triangles are joined along congruent sides.

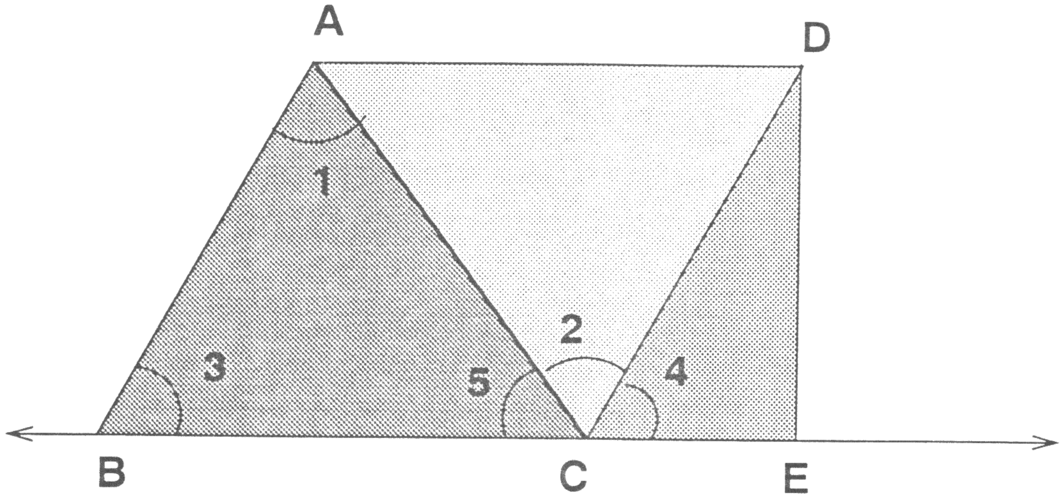

Fig. 3

Worksheet

1. Measure the following angles in the figure. $\angle 1 =$______, $\angle 2 =$ ______, $\angle 3 =$ ______, $\angle 4 =$ ______, $\angle 5 =$ ______.

2. Is $\angle 1 = \angle 2$? ____________. They are equal because __________. ________________________

   Ans: $\triangle$ABC and $\triangle$ACD are identical and $\angle 1$ and $\angle 2$ are the vertex angles of these triangles.

3. If $\angle 1 = \angle 2$, what can you say about the lines AB and DC?

   (Emphasize the converse reasoning here: because alternate angles are equal the lines are parallel)

4. Is $\angle 3 = \angle 4$? ______. They are equal because ________________ __________________________.

5. $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Hence $\angle 2 + \angle 4 =$ ______+ ______.

6. $\angle$ACE = _____________+ _____________. $\angle$ACE is called ________ __________________________.

7. *Hence the* _________________________*of a triangle is always equal to the* ______________*of interior* _________________________.

Note: In order to avoid possible misconceptions it is important to point out to students that there are two exterior angles possible at every vertex of a triangle.

Activity 4: To prove the S.S.S.congruency theorem for triangles.

Before starting on this activity the students should know the S.A.S congruency axiom. They should also know that angles opposite to equal sides of a triangle are always equal.

Collage work:

Cut two triangles in different colours, say one in red (△ABC) and the other in say pink (△DEF) such that they have all the corresponding sides equal. This can be done by choosing the length of the three sides and constructing the triangles using ruler and compass. Cut another triangle in blue (△GEF) such that two of its sides and the included angle are equal to that of the original red triangle. (In the figure AB = GE, AC = GF and ∠A = ∠G.) However, the blue triangle must be a mirror image of the red triangle. In other words, when the blue triangle is flipped upside down so that the white coloured surface of the glazed paper is on top, it must exactly cover the red triangle.

Glue the red and pink triangles separately. Glue the blue triangle (△GEF) inverted below the pink triangle (see Fig. 4). Draw a line (DE in the figure) joining the vertices of these two triangles.

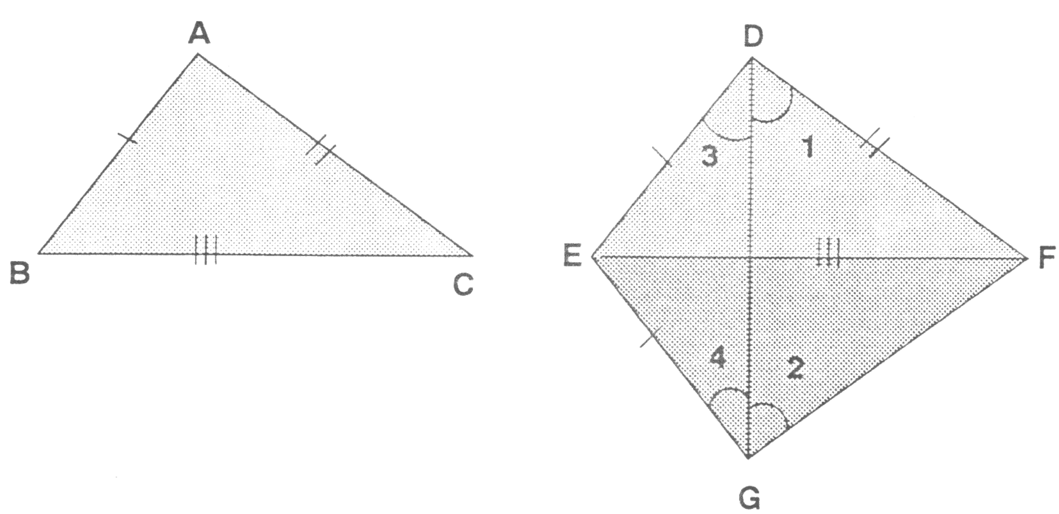

Fig. 4

Worksheet

1. Which pair of triangles do you already know to be congruent? Why are they congruent? ______________________________

2. Which pair of triangles should you now show to be congruent?
3. Compare the red and blue triangles. Is AC = GF? ____Why ? ______________________________
4. But AC = DF (given at the time of constructing the red and pink triangles), hence DF = ____
5. Similarly is AB = GE? ____, why? ________________. But by construction AB = DE. Hence DE = ____
6. What kind of triangle is $\triangle$DFG? ______________________. What can you say about the measures of $\angle 1$ and $\angle 2$? ______________________. Verify your answer by measuring the angles.
7. What kind of triangle is $\triangle$DEG? ______________________. What can you say about the measures of $\angle 3$ and $\angle 4$? ______________________. Verify your answer by measuring the angles.
8. $\angle$D = ____+ ____. $\angle$G = ____+ ____. What can you say about the measures of $\angle$D and $\angle$G? ________________
9. Now compare the pink and the blue triangles. In triangles DEF and GEF we have DE= ____, DF= ____and $\angle$D = ____
10. What can you say about these two triangles? ____________ ____________________. (Use S.A.S theorem)
11. We know that the red and blue triangles are congruent. We have proved that the pink and blue triangles are congruent. Hence the red and ____triangles are congruent. Or $\triangle$____= $\triangle$____

12. *Hence when three sides of a triangle are correspondingly ___________to the three sides of the other triangle then the ________________are ________________.*

Activity 5: To show that the bisectors of interior consecutive angles between a pair of parallel lines enclose a rectangle

Before this activity the students should know that the pair of alternate angles formed between two parallel lines are always equal. They should also know that sum of three angles of a triangle is equal to 180°.

Collage work:

Cut a rectangle in green paper (ABCD in Fig. 5). Cut two identical triangles one in red (△MPN) and the other in yellow (△MQN) such that they contain a right angle (∠P and ∠Q).

Glue these triangles on to the large green triangle such that their longest sides coincide (MN), and the vertices without the right angle fall on the edges of the green rectangle (M and N). Draw line l and line m through the common vertex.

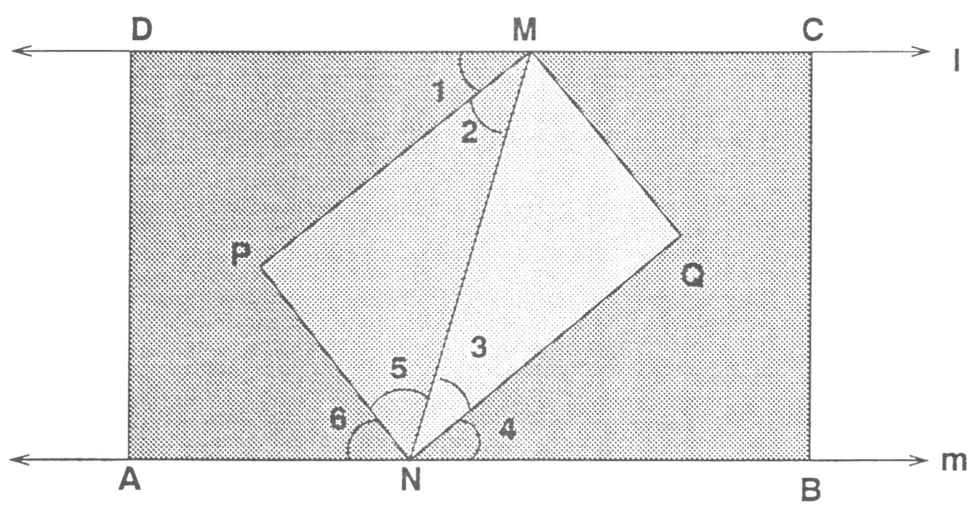

Fig. 5

Worksheet

1. Measure the angles in the figure. $\angle 1 =$ _____, $\angle 2 =$ _____, $\angle 3 =$ _____, $\angle 4 =$ _____, $\angle 5 =$ _____, $\angle 6 =$ _____.

2. Is $\angle 1 = \angle 2$? __________

3. What is MP called with respect to $\angle$DMN? ______________

   Ans: The line MP is the angle bisector of $\angle$DMN.

4. Similarly NP is the bisector of __________, MQ is the bisector of __________, and NQ is the bisector of __________.

   Note: Now using only the facts that the lines l and m are parallel and that MP and NQ bisect $\angle$DMN and $\angle$MNB, we must show that MPNQ is a rectangle.

5. Is $\angle 1 + \angle 2 = \angle 3 + \angle 4$? _____. Why would you expect them to be equal? _______. Is $\angle 2 = \angle 3$? _____. Why? _______.

6. What are $\angle 2$ and $\angle 3$ with respect to the transversal MN cutting the lines MP and NQ? ______________

   Ans: These angles are alternate interior angles.

7. Is MP parallel to NQ ? _____. Why? ________________

8. What is $\angle 1 + \angle 2 + \angle 5 + \angle 6 =$ _____? Why? ______________________________________.

   Ans: The sum of these two angles is equal to 180°, because these two pairs form interior consecutive angles between the parallel lines.

9. What is $\angle 2 + \angle 5 =$? __________, Why ? ______________

   Ans: The measure of $\angle 2 + \angle 5 = 90^\circ$, because $\angle 1 = \angle 2$, $\angle 5 = \angle 6$.

10. In $\triangle$MPN $\angle P + \angle 2 + \angle 5 =$ _____, because ______________.

11. Therefore $\angle P =$ _____.

12. Can you find the measure of$\angle$PNQ? $\angle$PNQ $=$ ______+ $\angle 5 =$ ______+ $\angle 5 =$ ______, because ______________________________
13. We have now to show that $\angle$Q is a right angle is similar to showing $\angle$P is a right angle. How would you do that? ______ If.
14. In quadrilateral MPNQ, we know that the measure of three of the angles is each equal to 90°. What is the remaining angle? ____________. What is its measure? ______. Why? __________.
15. Hence the quadrilateral MPNQ is a ________________________ because ____.
16. *Hence the bisectors of interior* ________________________ *angles between two parallel lines enclose a* ________________________.

Note: An alternative proof of the theorem is possible by showing MPNQ to be a parallelogram with one of its angles equal to 90°. In this alternative proof the students are also introduced to the interesting theorem that a parallelogram with one of its angles equal to a right angle is a rectangle.

Activity 6: To prove the Mid- Point theorem

In this activity the students prove that the line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

Before doing this activity the students should know the property of a linear pair of angles and the properties of parallel lines. They should also recall the congruence conditions for the triangles.

Collage work:

Cut any triangle from glazed paper in say red ($\triangle$ABC in Fig. 6). Draw a line through the mid-points of two of its sides (DE). A smaller triangle is formed ($\triangle$ADE) which has the same vertex as the original triangle. Now cut a triangle in say blue which is identical to the smaller triangle (△ADE). Two sides of the smaller triangle (AD and AE) are equal to half of the corresponding sides of the original triangle. The vertex angle (∠A) is equal in both the triangles.

Now cut a third triangle in say green (△CEF in the figure) which is identical to the smaller green triangle. Glue the blue triangle over the top of the red triangle with the vertex coinciding and the green triangle to the right of the red triangle, inverted as shown in Fig. 6.

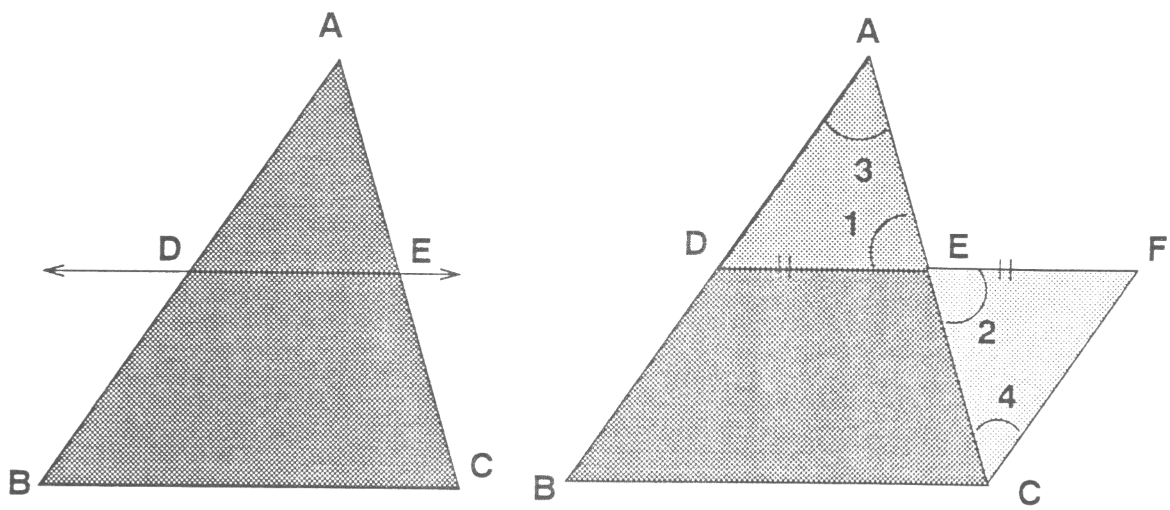

Fig. 6

Worksheet

1. Compare the two smaller triangles △ADE and △CFE. AE = ______, AD = ______, and ∠3 = ______.

2. What can you say about △ADE and △CFE? _______. Why? ________________________.

3. What can you say about the lengths of DE and EF? _________________. Why? _______________________________.

4. Similarly what can you say about the measures of $\angle 1$ and $\angle 2$ (Give reasons for your answer)? ____________________.
   Ans: The angles are equal because $\triangle ADE \cong \triangle CFE$. Note that we cannot use the relation between vertically opposite angles because we still do not know if D, E and F are in the same straight line.
5. $\angle 1 + \angle DEC =$ __________. Why? ____________________.
6. But $\angle 1 = \angle 2$. Hence $\angle 2 + \angle DEC =$ __________.
   Hence what can you say about the points D, E and F? _____.
7. What kind of angles are $\angle 3$ and $\angle 4$ with respect to the lines AD and CF which are cut by the line AC? _______________
8. Is AD parallel to CF? _____. Why? ____________________
   Ans: Since alternate interior angles are equal.
9. Is AB||CF ? __________. Why ? ____________________
10. So now we can conclude that BD is parallel to __________
11. Measure AD= __________, BD= __________, CF= __________.
    Why are they equal ? ______________________________
    Now in the quadrilateral BCFD, BD=CF and BD||CF. Therefore it is a _______________.
12. Is BC||DF ? _____, Why ? __________________________
13. Is BC = DF? _____, why? __________________________.
14. But DF = _____+ _____, and DE = EF. Therefore DE = 1/2 _____. What can you say about the relation between the lengths of DE and BC? ____________________.

15. What is DE with respect to the triangle ABC? ___________

    Ans: In triangle ABC, DE is the line segment joining the midpoints of two sides.

16. *Therefore in any triangle the line joining the mid-point of two sides is always ___________ to the third side and equal to ___________.*

Activity 7: To prove the intercept theorem

In this activity students prove that if the intercepts of a transversal cutting three parallel lines are equal then the intercepts on any other transversal are also equal.

Prior to doing this activity the students should know the properties of parallel lines and the Angle Side Angle (ASA) congruency theorem for the triangles.

Collage work:

Cut two rectangular strips of paper of equal width say one in orange and the other in blue. Glue them one below the other so that their edges meet and are parallel (the strips enclosed by the lines p, q and r in Fig. 7).

In the same way cut a slightly wider strip in say yellow (enclosed by the lines l and n in the figure.) The yellow strip is pasted across the two parallel strips so that its two edges form transversals (DF and AC) cutting the first two parallel strips. Now cut a narrower strip in say red (enclosed by the lines l and m in the figure). The red strip is pasted over the yellow strip so that all four strips intersect at a point E as shown in the figure. Draw the lines p, q, r, l, m and n along the edges.

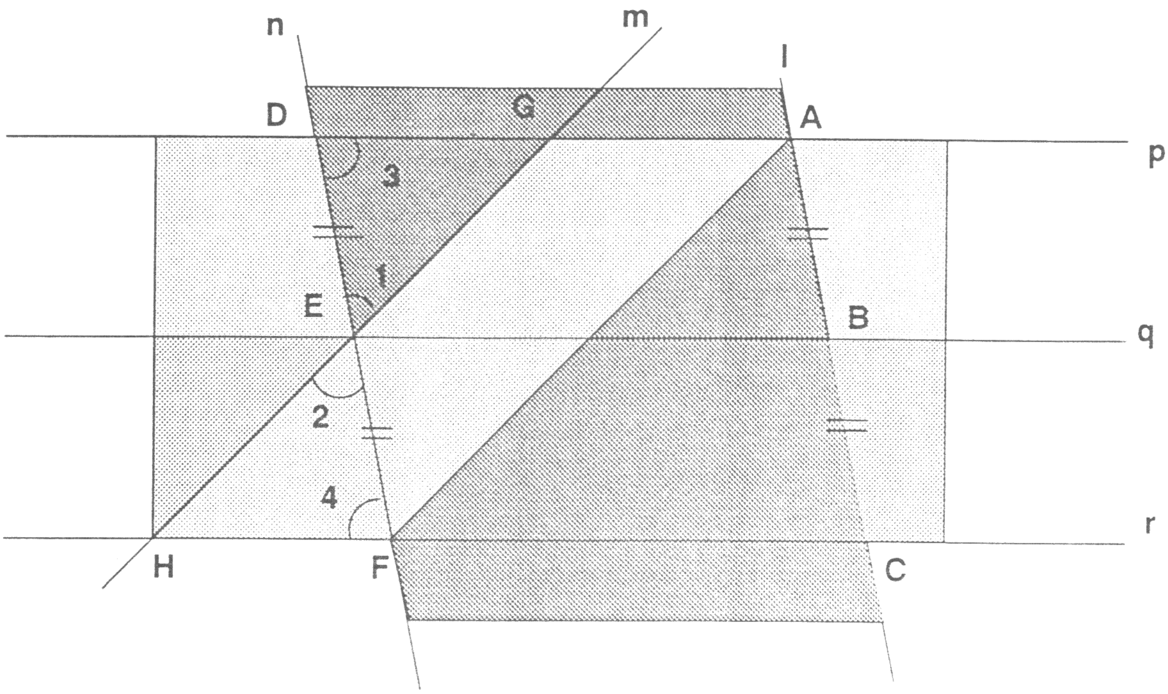

Fig. 7

Worksheet

1. Measure the lengths AB and BC. AB= __________and BC= __________.

   Note that these lengths are equal. This is taken as our starting condition.

2. We know that p||q and q||r by the construction. Similarly can we say n||l by construction? ____________________.

3. Hence in the quadrilateral ADEB opposite sides are __________ to each other. Therefore it is a ____________________.

4. Is the quadrilateral BEFC also a parallelogram ? _____.
   Why? ____

5. In the parallelogram ADEB, is AB = DE ? __________.
   Why? __________________________

6. In the parallelogram BEFC, is BC = EF ? __________.
   Why? __________________________

7. We know that AB = BC. Hence what is the relation between the lengths of DE and EF? _____________________.

8. Measure the angles in the figure. $\angle 1=$ ______, $\angle 2=$ ______, $\angle 3=$ ______, $\angle 4=$ ______.

9. Why is $\angle 1 = \angle 2$? __________________________. Why is $\angle 3 = \angle 4$? __________________________

10. Is $\triangle$GDE congruent to $\triangle$HFE? ______. Why ?______________
    Ans: The two triangles are congruent because two angles and the included side of one triangle are correspondingly equal to those of the other(ASA).

11. Is GE = HE ? ______, why ? __________________________

12. *Hence between three parallel lines when the intercepts on one* ______________________*are equal, then the* ______________________*on any other* ______________________ *are also equal.*

Activity 8: To prove the median concurrency theorem

In this activity students prove that the medians of a triangle intersect at a point, and divide each other in the ratio 2:1.

For this activity the students should know the property of a parallelogram, including the property that the diagonals bisect each other and recall the midpoint theorem of activity 6.

Collage work:

Cut a triangle ABC in say red and draw two of its medians AD and BE. Name their intersection point as G, join CG. Cut a parallelogram in blue colour such that the length of the two consecutive sides of the parallelogram should be equal to BG and CG.

Glue the parallelogram over the triangle so that two consecutive sides of the parallelogram coincide with BG and CG. Thus three of its vertices coincide with B,C and G respectively. Name the remaining vertex K. Now glue a thin yellow strip from A to D through G, and a fourth red strip from B to C.

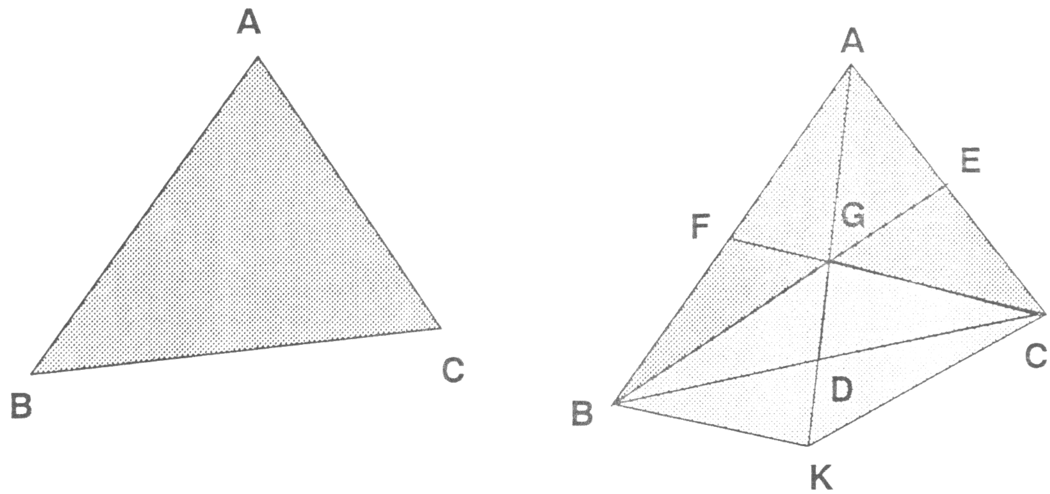

Fig. 8

Worksheet

1. In the parallelogram BKCG, GK is one of the diagonals. Does it pass through D? _____.
   Why? ______________________________

2. Hence G, D and K lie on the same __________________.
   Produce GD to pass through K.

3. In $\triangle$ACK, is GE parallel to KC? _____. Why? ____________
4. We know that E is the mid-point of AC. Is G the mid-point of AK? _____Why? ___________________________

   Ans: In a triangle the line parallel to the base passing through the midpoint of a side passes through the midpoint of the other side
5. GD is half of GK, because _____________________________
   But GK = _____. Hence GD = 1/2 _____.
6. *Hence the point G divides the median AD in the ratio* _____.
7. We have to prove that the third median from C to AB also passes through G. We will do this indirectly. First produce CG to meet AB in F.
8. Consider $\triangle$ABK. Is FG||BK? _____Why?________________
9. We know that G is the mid-point of AK. Is F is the mid- point of AB? _____Why? ________________________________
10. Hence CF is the _______________from C to AB. We already know that it passes through G. *Hence the* __________*of a triangle intersect* ___________________________.
11. We have shown that the median AD is divided in the ratio 2:1 at the point of intersection G. How will you show that this is also true for the medians BE and CF? ___________________
12. *Hence the medians in a triangle pass through* __________*point and divide each other in the ratio* __________

Activity 9: The area of parallelograms between same parallels and on the same base are equal.

Before doing this activity the students should know the area addtion axiom and should recall that congruent triangles have equal area.

Collage work:

Cut any non-isoceles trapezium (ABCD in Fig. 9) from say pink coloured paper. Now cut a smaller trapezium (DEFC in the figure) in orange such that its longer parallel side (DC) is equal to the shorter parallel side of the pink trapezium (DC). Also its non-parallel sides must be equal to the non-parallel sides of the pink trapezium (AD = CF and BC = DE). When the two trapezia are aligned upright with the smaller inside the larger, the non-parallel sides which are equal must lie adjacent to each other. Now glue the smaller trapezium inside the larger trapezium in inverted fashion as shown in the figure. The sides which are equal form boundaries of two parallelograms (ADCF and BCDE) as seen in the figure.

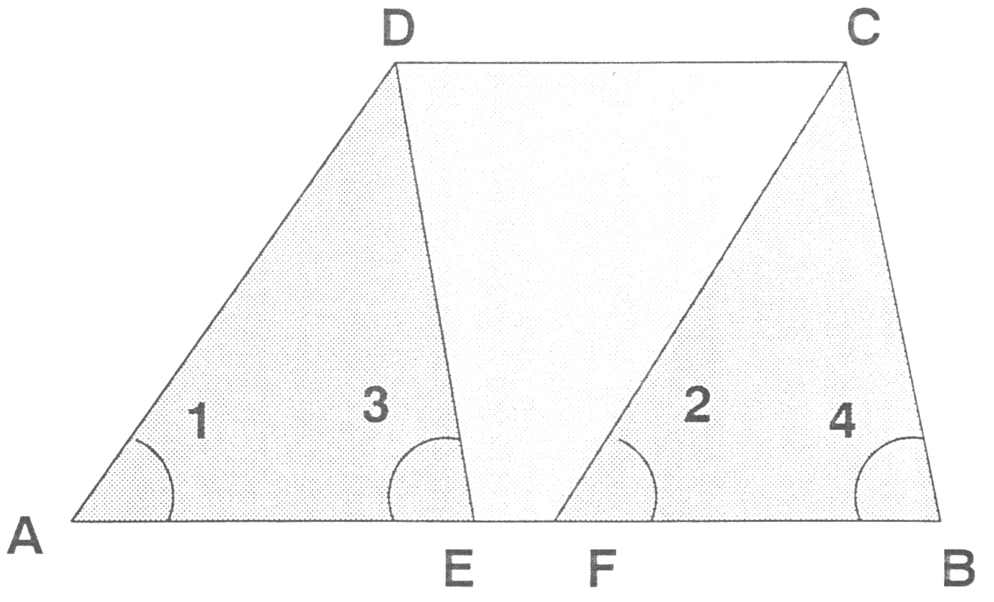

Fig. 9

Worksheet

1. Observe that there are two parallelograms in the figure. They are ___________and ___________. Are these parallelograms congruent? ______. Can you say that their bases are equal? ______. Do they lie between the same parallel lines? ______.
2. Area of parallelogram ADCF = area of quadrilateral ______+ area of triangle ___________
3. Area of parallelogram BCDE = area of quadrilateral ______+ area of triangle ___________
4. Is △ADE congruent to △FCB? ______Why?________________

   The students may use the property of parallel lines and corresponding angles or the S.S.S. theorem.
5. Is area of △ADE = area of △FCB? ______Why? ___________.
6. What is the area common to the two parallelograms? ______.
7. Is the area of the two parallelograms equal ? ______.
   Why? ____

   Ans: Since the two triangles in the two parallelograms have equal area and there is a common region, their areas are equal.
8. *Hence the* ___________*of two* ____________________________*on the same base and between* _______________________*are always* ___________

Note: It is necessary to point out we have assumed that DE and CF do not intersect. Let the proof of the theorem with the lines DE and CF intersecting be given as an exercise. This calls for a slight modification of the proof, but the approach remains the same.

Activity 10: To prove the Alternate segment theorem

In this activity students prove that the angle made by a chord with a tangent in a circle is equal to the angle in the alternate segment.

Before performing this activity the students must know that the angles in the same segment of a circle are equal. They should recall the following: radius is perpendicular to the tangent at the point of contact, sum of the angles in a linear pair is 180°, opposite angles in a cyclic quadrilateral are supplimentary.

Collage work:

Cut a circle from glazed paper in say red. Draw the circle with a divider instead of a compass and the circle can be cut with one of its sharp points.

Cut a right angle triangle ($\triangle$PBA in Fig. 10) with its hypotenuse equal to the diameter (PA) of the circle. Cut a second triangle ($\triangle$PBC in the figure) which is not right-angled such that (a) its base (PB) is equal to one of the sides containing the right angle of the first triangle (PB) and (b) all three vertices (P, B and C) of the triangle fall on the circle and (c) the vertex angle ($\angle$C) is acute.

Now cut a third triangle ($\triangle$PDB in the figure) such that its base (PB) is equal to the base of the second triangle (PB) and its obtuse vertex angle ($\angle$D) is supplementary to the vertex angle ($\angle$C) of the second triangle.

Glue all the triangles over the circle and draw the line AP through the centre O and RPQ through P perpendicular to AP.

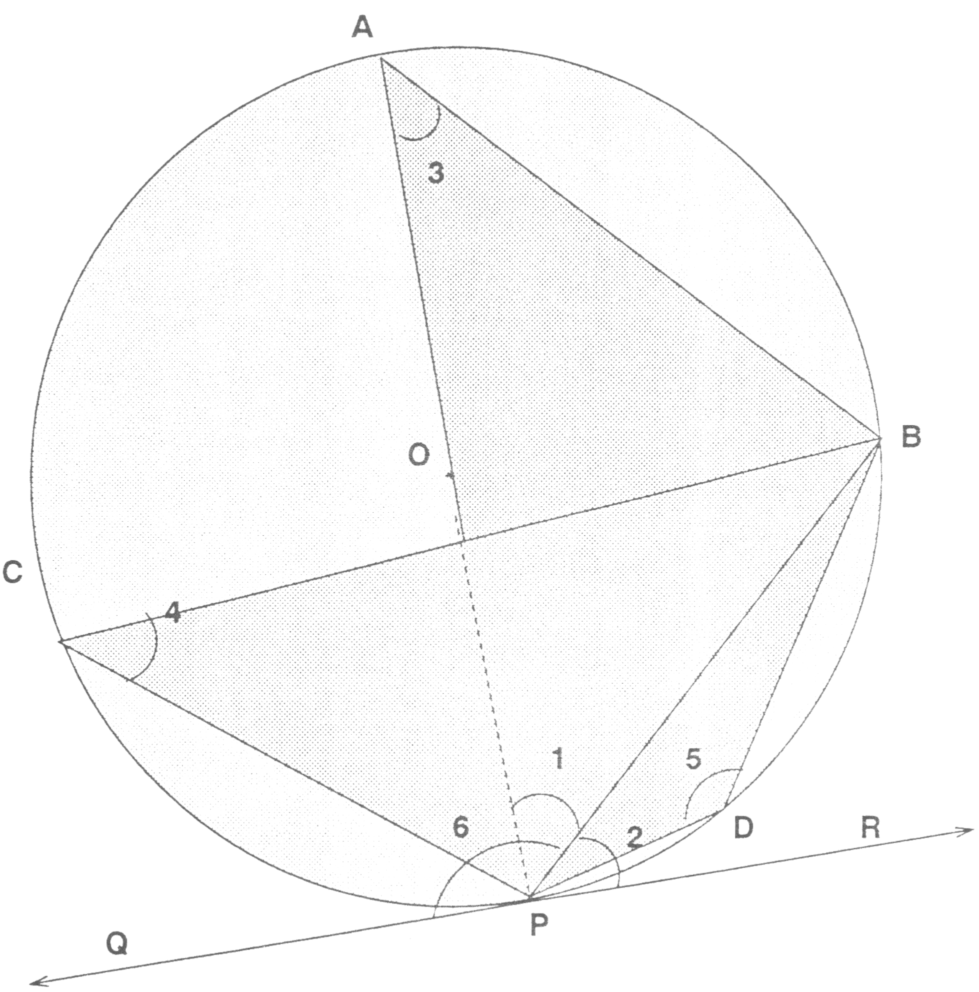

Fig. 10

Worksheet

1. What is the line RPQ called? ____________________

2. Measure the angles in the figure. ∠1= _____, ∠2= _____, ∠3= _____, ∠4= _____, ∠5= _____, ∠6= _____

3. ∠ABP =_____, because ____________________

4. ∠1 + ∠3 + ∠APB = _____. Hence ∠1+ ∠3 = __________

5. What is $\angle 1 + \angle 2 =$ ______? Why? ________________________

6. Is $\angle 1 + \angle 3 = \angle 1 + \angle 2$? ______. Therefore $\angle 2 =$ ______.

7. What can you say about the measures of $\angle 3$ and $\angle 4$? Give reasons for your answer. __________________________________

   Ans: These angles are equal because they are in the same segment of the circle

8. Therefore $\angle 2 =$ ______ $=$ ______.

9. $\angle 4$ is the angle in the __________________ segment with respect to $\angle 2$.

10. What is the quadrilateral BCPD called ? ________________________.

11. In quadrilateral BCPD $\angle 4 + \angle 5 =$ ______, because __________

12. $\angle 2 + \angle 6 =$ ______, because _______________________________

    Thus we can say that $\angle 2 + \angle 6 = \angle 4 + \angle 5$, since their sum is $180°$.

13. But we have seen that $\angle 2 = \angle 4$. Hence ______ $=$ ______

14. $\angle 5$ is on the __________________________________ with respect to $\angle 6$.

15. *Therefore the angles made by the tangent with a* ____________ *is always equal to the angle in the* ______________________________.