Understanding teachers' mathematical knowledge using non-typical examples

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Mathematical knowledge for teaching is the area researched by many researchers across countries. Effective and good teaching is dependent on teachers' own understanding of mathematics is well understood in the field. At HBCSE, with experience of several years of in-service teacher education programme, we conceptualize the nature of teachers' knowledge as being composed of two major areas (teachers' content knowledge and learning pedagogic techniques), found similar to what other researchers have proposed with some cultural variations. The question which I try to address here is what are the tools available for us to develop teacher's content knowledge. I discuss one of the tool as use of non-typical examples. How do they work and bring changes in teachers' cognition? Do such examples facilitate or impede learning? I discuss here some examples that I used in teacher education workshops and try to understand the learning that occurred during interviews and discussions with the teachers, which was conducted subsequent to their written response to these examples.

Introduction

Concern about students' learning of mathematics has directed the attention of everyone towards the kind of mathematics flowing in the classroom, which in a typical Indian classroom has been originated from teacher. Hence teachers' understanding of and about mathematics becomes the crucial part of mathematical content of the classroom. Lamper (2001) addresses the classroom teaching proceeds simultaneously in relation with students, with content and with connection between student and content. The third part is a more complex part which demands connection between students and content, students responses and right places for these responses for contribution to the content, progress in students' content and at the same time increase in complexity of the content in the classroom itself, and many social issues of the classroom. Observation of mathematics classes suggests that teachers' knowledge of mathematics and their ability to deploy it in teaching, matter for the quality of students' opportunities to learn (Ball, et al 2004). But still what constitutes 'knowledge of mathematics for teaching' is not commonly defined and according to me it has many parameters. The domain of teachers' knowledge identified by Shulman (1987), which he termed as 'Pedagogic Content Knowledge', made the distinction between knowing the content for 'oneself' and knowing it with pedagogy required for teaching this content. This idea by Shulman focused teacher education on the content knowledge required for teaching. This is not in contradiction to what Dewey said that content is not separate from its method of explanation, better said content contains the method of explanation. But inadequate knowledge of the concepts can give rise to inadequate methods of explanation (pedagogy). Ball (2007) points out that there is much more to the" Pedagogic content knowledge" than just to refer it to a wide range of aspects of subject matter knowledge and the teaching of subject matter, but the potential of the term remains insufficiently exploited.

In case of teachers in India it is understood that none of the educational courses give opportunity to understand the content required for their own teaching (Naik, 2008; NCTE, 2006). It seems more emphasis on methods of teaching with out considering what are we going to teach.

According to this view, mathematical knowledge for teaching goes beyond that captured in measures of mathematics courses taken or basic mathematical skills. For example, teachers are not expected to only calculate correctly but also to be able to justify each and every derivation with possible representation. How is this knowledge attained? As Ma (1999) describes Profound Understanding of Fundamental Mathematics (PUFM) is attained in Chinese teachers in their pre-teaching courses and in actual teaching careers by following means - (a) studying teaching material intensively, (b) learning Mathematics from colleagues, (c) learning Mathematics from students and (d) learning Mathematics by doing it. So this gives us insight that, the knowledge of mathematics which is tailored to the work teacher do with curriculum materials, instruction and students is attained by doing activities pertaining to the profession of teaching. In HBCSE, we[1] have also developed some examples to work with teachers and know more about their knowledge of mathematics. I am trying to make an attempt to see teachers thought processes in attempting such mathematical problems meant for checking their mathematical knowledge for teaching.

Theoretical framework of teacher workshops at HBCSE

The experience of in-service teacher education at HBCSE has led us to conceptualize teachers' knowledge as being composed of two major domains - content knowledge and knowledge of pedagogic techniques (See fig. 1). Here the term 'Pedagogic techniques', is

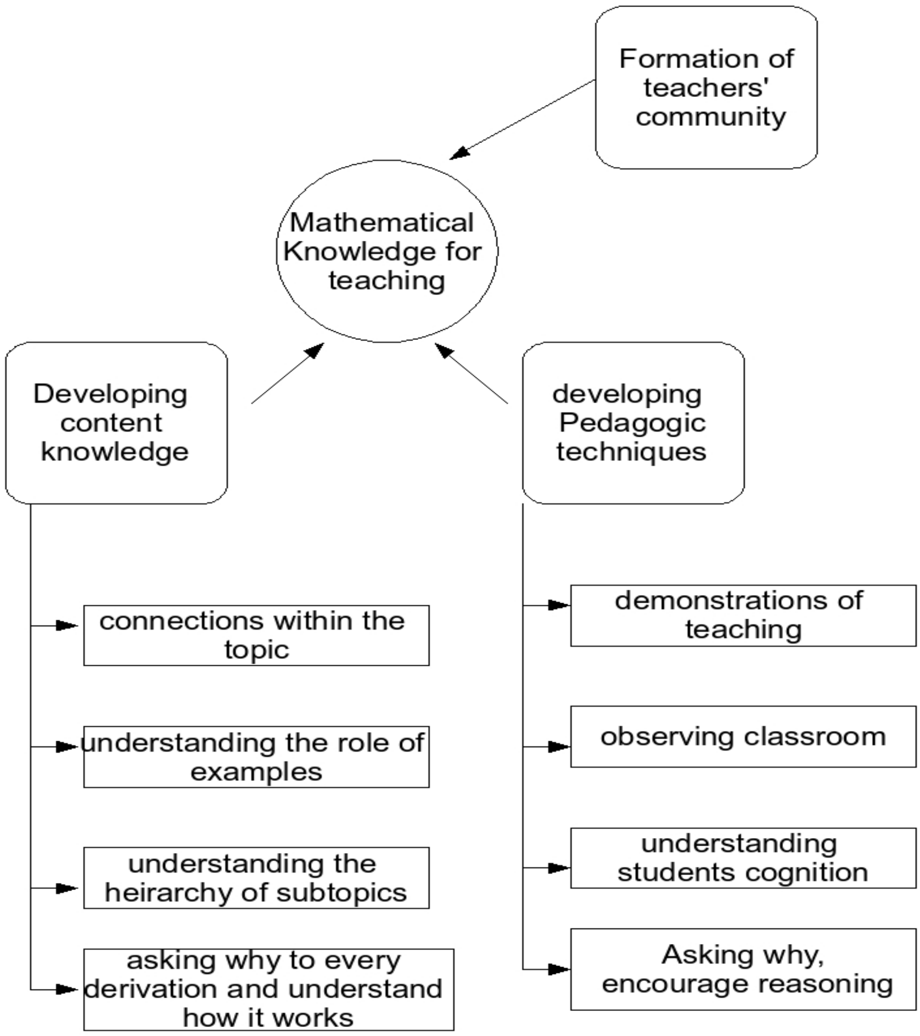

fig. 1: Domains of Teachers' knowledge

used to explain the association between knowledge of a concept and the instruction/ demonstration required for its delivery considering resources available in typical classroom. The workshops we conduct for teachers talk about the techniques which are non-subjective that is independent of the individual teacher but particular to the subject of mathematics.

Even though the picture above gives the brief idea of the format of teacher education at HBCSE, it would take a long discussion to actually explain it. Let me discuss our approach to teacher education programmes. In the beginning of any teacher education programme we expose teaches to the non-typical examples. From a mathematical perspective, an example must satisfy certain mathematical conditions depending on the concept or principle it is meant to illustrate; from a pedagogical perspective, an example needs to be presented in a way that conveys its 'message', specially when you use it as an assessment. In spite of the above, most mathematics teacher education programs do not explicitly address this issue. We ** ate HBCSE have tried to develop collection of some mathematical problems which we use in Teacher education programmes. An important characteristic of these examples is they are constructed on common misconceptions and the format of the problem allow solver to reach towards the conflict and to resolve it on her own.

Analysis of teachers work on some problems

Teacher education and specially assessment of teachers content knowledge requires some care which we take during adult education. Hence it puts limits on the way questions can be asked to the teachers about their own understanding of the content. We ask questions in the format which they are very familiar with. The set of examples in the beginning of teacher education workshops includes four or five solutions to each of the question given as if they are the responses given by different students and teachers need to check each of the solution whether it is done right or wrong. An example of such question is as follows-

In the following question answers given by students are given as options. Check each option whether it is right or wrong.

7 2/5 – 7 × 2/5 = _____________

a) 0 b) 2/5 c) 4 3/5 d) 23/5

In the above example, 68% of teachers marked option (a) as a correct answer. This data is from one of our teacher education workshop, but the result was not very surprising for us from our experience with other teachers. Such questions available with multiple response challenge the teachers' knowledge and beliefs about the concepts involved. Many teachers have learned the conversion of mixed number into fractions as multiplication of the whole number and the denominator followed by addition with the numerator. This procedural understanding may develop a belief of the existence of a multiplication sign between 7 and 2/5. The existence is also supported by the rules from algebra as it is often said that if there is no sign between two letters (or a letter and a number) then there is a multiplication sign. So *xy indicates that* $x \times y$. *Similarly*, 7 2/5 indicates 7 × 2/5. Such generalisation may also have lead those teachers towards wrong answer i. e. option (a) 0.

Seeing their response, in the interview session I asked them to explain how did student arrived to other set of answers that is (b) 2/5, (C) 4 3/5 and (d) 23/5. They started giving possible thinking that student's might have done. This unpacking of what students thought, gave them insight about the structure of fraction notation itself. One can interpret the notation differently and hence sometimes wrongly was recognised by some of them.

The interview with teachers showed that they knew how to carry out the multiplication of fractions or fractions with the whole numbers (numerator × numerator/ denominator ×denominator*)*. They also knew procedurally how to convert a mixed number to the fraction form. While explaining option (C) teachers arrived to the conflict. This conflict created the need to understand the relationship between the procedure and meaning of the procedure. The task gave them the platform to change their representation of fractions. For example, one teacher who had earlier made the error of equating 7 2/5 – 7 × 2/5 to zero, argued as follows -

7 2/5 = (7 ×5 + 2) / 5

= 37/5

= 35/5 + 2/5

= 7 + 2/5 = 7 2/5

The derivation above was a rediscovery for that teacher as she proved that there is no multiplication sign in 7 2/5 but 7 and 2/5 has operation of addition in between them. For me above derivation was like giving a small arithmetic proof of 7 2/ 5 = 7 + 2/5. The teacher above who gave wrong answer in the beginning got the opportunity to correct herself through the help of an example and its form of presentation. Such opportunities may not be available for teachers in the traditional textbook assessment questions. Also re-teaching the concept of fraction to teachers might not create any challenges to the existing knowledge of theirs. But an example such as above gives them the platform to challenge their own understanding, repair it and reform it.

Let us see one more example -

> *In the following question answers given by students are given as options. Check each option whether it is right or wrong.*
>
> *Find GCD of 8 and 9?*
>
> *72 (21%)*
>
> *0 (56%)*
>
> *1(8%)*
>
> *There is no method for this calculation (12%)*

The percentage in bracket shows how many teachers marked that particular option as a correct solution. The result is very shocking, but if one analyses it in detail we see that this wrong understanding has emerged from some unwanted over generalisations. 72 is LCM of 8 and 9, which is hurriedly understood and lets say was mistakenly marked. What happened in option (b), following is the response from a teacher "*For GCD we need common divisors, there is nothing common in 8 and 9, and nothing means zero.*" Why 1 as a common divisor was missed out, may be because in unique prime factorization, we don't use 1 as prime factor. This example is different from the above, but we see the interference of the rules and language, formed for the purpose of saving time in other topics of school mathematics even in a straight forward problem like this. We teach nothing means zero, but "nothing" is with the context. So no other common divisor automatically brings us to only prime divisor of any number which is 1. Such numbers (like 8 and 9) are called as co prime numbers.

Conclusion and comments

We have many such examples, which can be used in teacher education programmes for the purpose of content development. In every teacher education programme, teachers work first individually on these examples and then discuss with me or in groups reasons for all the solutions given for each example. The illustration above answers a question of how one can approach for development of coherent understanding of the concepts among teachers. The re-teaching of any topic may not bring forward their conflicts and wrongly developed beliefs, which takes the methodology described above at the centre to any teacher education programme.

At HBCSE the group working in Mathematics Education is researching along many parameters through which teacher development is possible. In the paper above I tried to describe the role of examples that we use for the purpose of content development of teachers. Questions on other developmental issues such as developing pedagogic techniques, understanding coherent sequence of a topic or development of teachers community for sharing resources are welcomed. You can write to me on the email address given above.

Note:

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