Students' Use of Language and Symbols to Reason about Expressions

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Reasoning and explanation by students has been of interest to most mathematics and science educators and psychologists like Piaget. Reasons are used to justify propositions and involve the use of language and symbols (mathematics or otherwise). Researchers have explored the use of language and symbols (signs and drawings) by students in the process of reasoning (Robotti, 2002; Radford, 2001). Vygotsky has also considered language as a tool for the construction and management of thinking.

Traditional mathematics classrooms, which are mostly teacher directed, emphasize writing clear steps, using correct symbols, syntax, etc., and not thinking about these objects, identifying relationships between these objects and operation signs and giving reasons using language or symbols. A number of studies have pointed out students' inability to deal with symbolic expressions. Students can neither make sense of the structure of the expression nor consistently use rules to manipulate them successfully (Kieran, 1989; Chaiklin and Lesgold, 1984; Linchevski and Livneh, 1999; Liebenberg, Linchevski, Sasman and Olivier, 1999). In a study conducted at the Homi Bhabha Centre for Science Education, Mumbai we have tried to capitalize on students' intuitive understanding of mathematical symbols and expressions, where they use their own language as well as symbols to reason about expressions. This work is part of a larger study, which focuses on the transition from arithmetic to algebra (Subramaniam and Banerjee, 2004). It is a teaching intervention study, being conducted on sixth grade students (11 to 12 year olds) in three phases. In the 2^{nd} and 3^{rd} phase of the study, a batch learning in the local language Marathi was included. One of the goals of this study is to let students focus on the structural aspect of arithmetic expressions and to exploit their structural understanding in the learning of algebra. Through the instructional modules, students learn to see expressions as a number as well as a relation (for e.g., 3 + 2 stands for a number which is 3 more than 2). The structural understanding of the expression emphasises the concept of 'term' as a structural element and the concept of 'equality' as a structural relation between expressions.

In this poster presentation we will discuss students' use of language and symbols as a tool to justify their responses in three kinds of tasks: (a) exercises on comparing expressions without calculation, (b) finding the value of an expression given the value of a related expression and (c) filling in the blank with a term to make two given expressions equal. These tasks were given to students to consolidate the idea of terms and to reinforce correct parsing of expressions. Students had to justify their answers for each of the these questions using language or symbols. Reasoning enabled students to make explicit their thinking about mathematical objects as well as compelled them to look for relationships among these objects. Students, in the beginning, find it easier to verbalise their justification using language. But gradually some students spontaneously start using symbols to explain their argument. Others may start using this strategy taking the cue from their peers or the teacher.

In the tasks on comparing two expressions without calculation there were three subtypes: (i) expressions with one term constant (e.g. 37 + 58, 36 + 58), (ii) expressions involving terms compensating each other completely, making them equal (e.g. 53 + 38, 54 + 37) or (iii) expressions with partially compensating terms (e.g. 53 + 38, 55 + 37). Similar subtypes were posed for expressions with negative terms. The performance in all the subtypes of tasks involving only positive terms was high for all groups of students (80% to 90%). For exercises with negative terms, there was a difference in the percentage of correct responses between the groups that had been exposed to integers, albeit in a different context (80% to 90%), and the groups not exposed to integers (60% and 80%). The number of students who justified their answers with reasons is around 50% to 70% depending upon the complexity of the subtype tasks.

In the tasks on finding the value of an expression given the value of a related expression, there were two subtypes. One of the subtypes involved arithmetic expressions (if 326 + 598 = 924, then 324 + 598 = ?) and the other involved algebraic expressions (if $y + 35 = 72$, then $y + 34 = ?$). 65 % of the students successfully found the value of the related expression in the subtype involving arithmetic expressions and 56% of the students could justify their answer by giving a reason. In the second subtype, involving algebraic expression, 52% of English medium and 68% of Marathi medium students found the value of the related expression. The percentages of students giving reasons for the second subtype is 26% for English medium and 52% for Marathi medium students.

In the task of filling in the blank by a term to make two expressions equal (35 + 29 = 35 + 27 ___), 45% of the English medium students and 77% of the Marathi medium students accomplished the task successfully. Around 20% of the English medium and 5% of the Marathi medium students wrote the sum of the expression at the left side or the right side or sum of all the numbers in the blank (For e.g. 35 + 39 = 35 + 27 $\underline{+ 64}$). Around 10% of the students wrote $\underline{-2}$ in the blank, probably using '=' as a sign for association indicating that 27 is 2 less than 29.

In the above three tasks, students gave reasons to justify their answers using language or symbols depending upon the subtype. In the 'compare two expressions without calculation' task, most students used only language while reasoning for the simple tasks where the pair of expressions had one common term. For subtypes involving complete or partial compensation, students often used both language and symbols to justify their answer. Some students compared numbers and some compared terms; some students compared the value of the expressions as a whole. Some students found the difference of the differences between terms in expression-pairs with partially compensating terms and even wrote out their reasons using symbols as in this example: "*28 (+1) + 32 (-2) < 27 + 34*". For the question if $y + 34 = 72$ then $y + 34 = ?$, one of the responses was, "If you add 35 to y you get 72 therefore if you add 34 to y you will get 71". In the poster presentations, a variety of such responses will be described.

The responses to these tasks show that students have the ability to spontaneously use language to make sense of the expressions and transformations on them. Some of them could also subsequently use symbols to show the transformations on the expressions. Looking at the students' ability to use language and subsequent use of symbols by some students in the process of reasoning, we can hypothesize the transition from intuitive to language-based to symbolic reasoning as a way to make sense of symbolic expressions and syntactic transformations on them.

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