Geo-metry
MEASURING THE EARTH BY SHADOWS
'Geo-metry' literally means measuring the earth!

How to measure the diameter of the earth:

Find two places A and B on the same longitude (like Bhopal and Bangalore). Let the distance between the two places be x km.

On a pre-arranged date mark the path of the tip of the shadow of a vertical stick at both places. From the path find the shortest shadow which occurs exactly at noon, local time.

Find the angle θ between the sun's rays and the vertical stick.

$$\theta = \tan^{-1} (\text{length of shadow '}l\text{'} / \text{height of stick '}h\text{'})$$

The arc of longitude between A and B makes an angle α at the centre of the earth. α is either the sum or the difference of θ_A and θ_B (see figure 1 and figure 2).

The angle that you traverse when you move from A to B is α and the distance is x kms. If you traversed 360°, you would have travelled a distance equal to the circumference of the earth.

$$\alpha \quad : x \text{ km}$$
$$360^\circ : ? \text{ km}$$

Assuming that the earth is a sphere,

$$\text{circumference} = \frac{360 \times x}{\alpha}$$
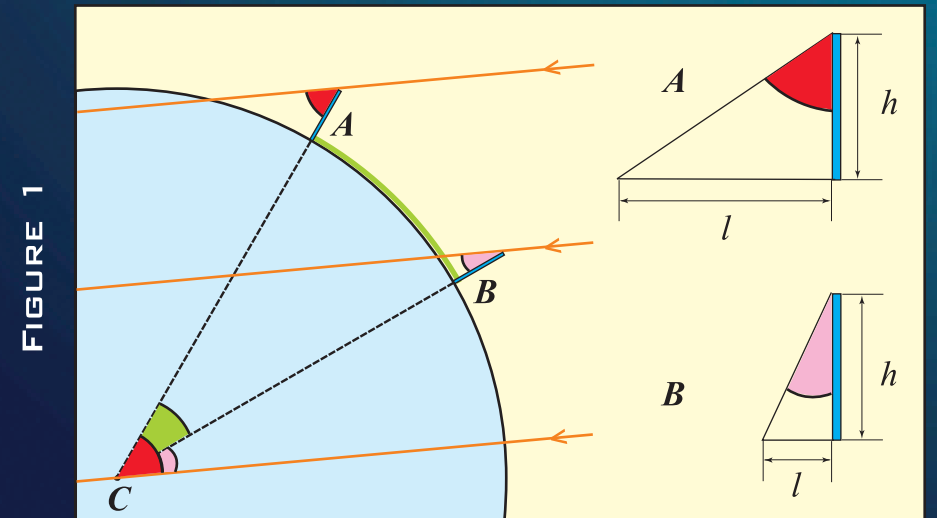$$\text{diameter} = \text{circumference} / \pi$$

FIGURE 1

If the shadows at A and B point in the same direction (i.e., both to the North or both to the South), then the arc of longitude between A and B makes an angle $\alpha = \theta_A - \theta_B$ at the center of the earth.
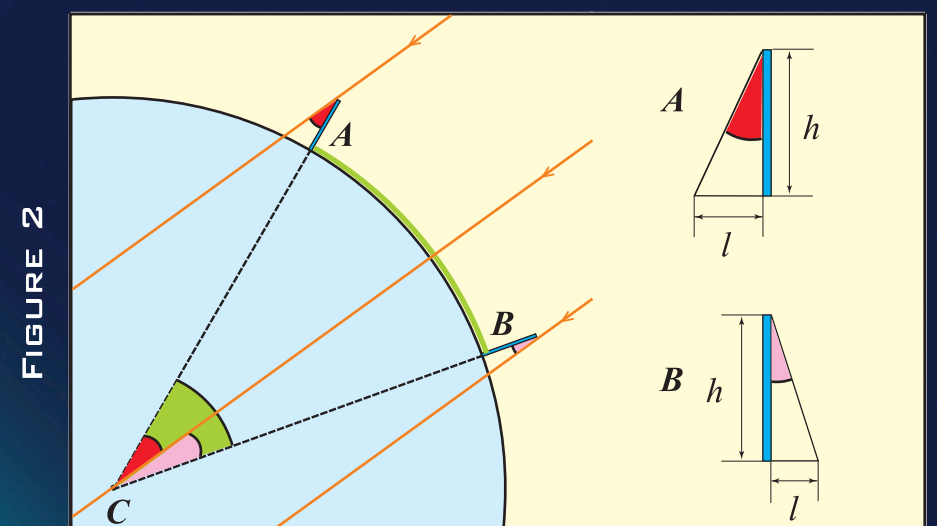

FIGURE 2

If the shadows point in opposite directions, the arc of longitude between A and B makes an angle $\alpha = \theta_A + \theta_B$ at the center of the earth.

Caution: *Not to scale. The height of the stick is greatly exaggerated.*

• Bhopal

• Bangalore

The diameter of the earth was estimated using roughly the same method in 250 BC! ***Eratosthenes***, *who lived in Alexandria in Egypt found the diameter to be 13,200 km (the error is only about 3.5%).*