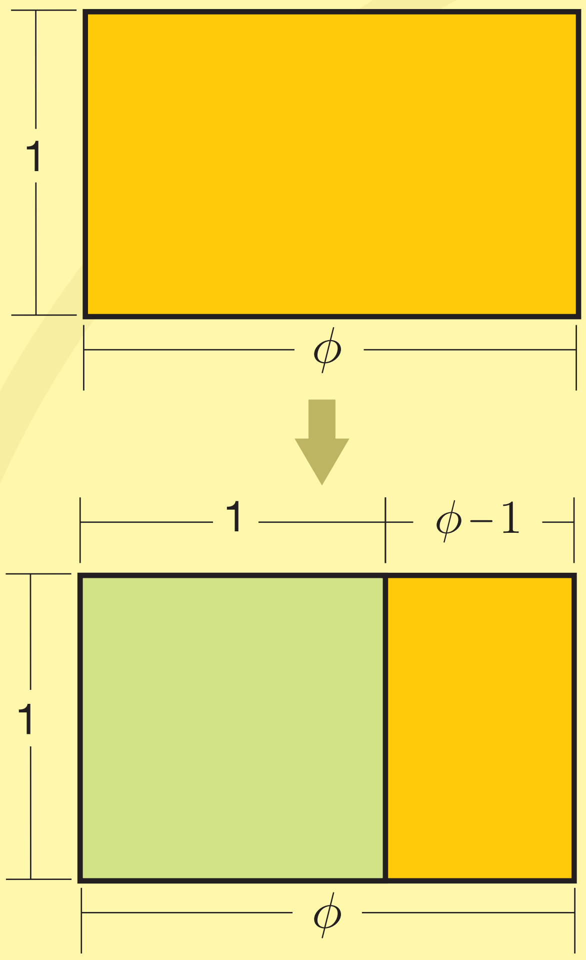
Golden ratio

Golden Rectangle



This is a rectangle whose sides are in the golden ratio.

When we remove the biggest possible square from this rectangle, the remaining part is also a golden rectangle.

The two golden rectangles in the figure are similar.

So, $\frac{\phi}{1} = \frac{1}{\phi - 1}$ (Equation 1)

Rearranging, we get $\phi^2 - \phi - 1 = 0$

Therefore, $\phi = \frac{1+\sqrt{5}}{2} = 1.6180339887...$

From equation 1, we also get $\phi - 1 = \frac{1}{\phi}$

$$\phi = 1.61803398874989484820458 68...$$

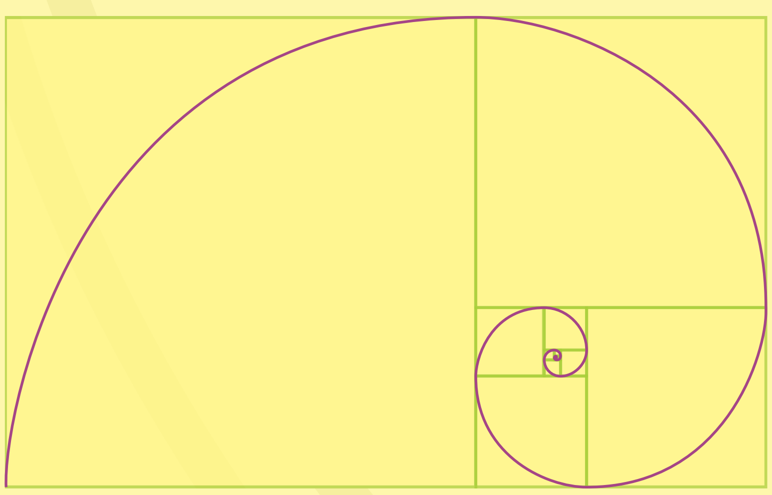
$$\frac{1}{\phi} = 0.61803398874989484820458 68...$$

We also see that, $\phi = 1 + \frac{1}{\phi}$

$$= 1 + \frac{1}{1+\frac{1}{\phi}} = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{\phi}}} = ...$$

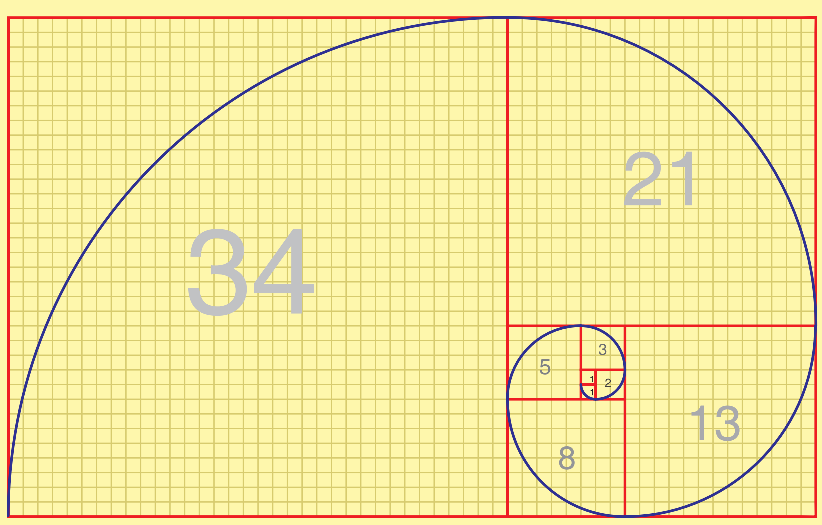
Golden Spiral

When we cut away squares repeatedly from a golden rectangle, we see that the golden spiral touches the opposite corners of each square.

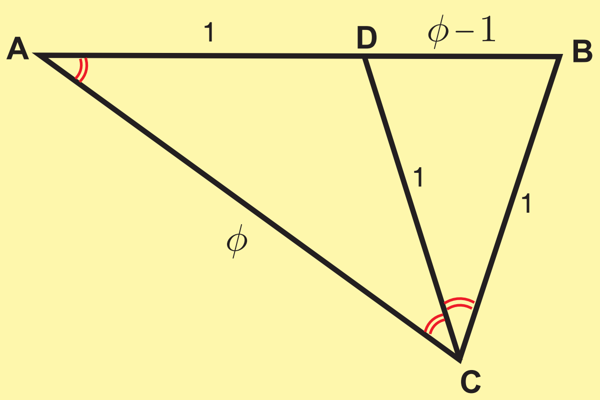


Fibonnacci Spiral

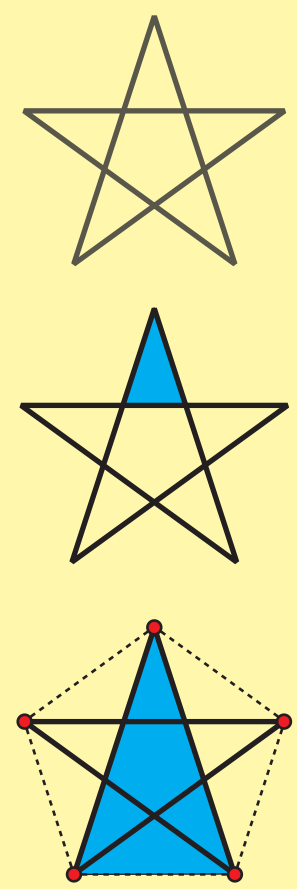
The sides of all the squares in the figure below are Fibonacci numbers. The Fibonacci spiral consists of quarter circles passing through opposite corners of each square. It approximates the golden spiral because the ratio of consecutive Fibonacci numbers converges to the Golden ratio. Unlike the Golden spiral, it terminates in the penultimate square.



Golden Triangle



The isosceles triangle ABC is a golden triangle whose sides are in the golden ratio to the base. We mark D on AB where CD=1. This makes ΔCDB an isosceles triangle similar to ΔABC. (Can you prove this?)
So ΔCDB is also a golden triangle. From this it follows that $\angle A = \frac{1}{2} \times \angle B = 36°$. (Can you prove this?)



The pentagram which is a very beautiful symmetrical shape found in ancient art and religion is full of Golden triangles. Try to find as many golden triangles as possible in the pentagram above. Two are already shown.