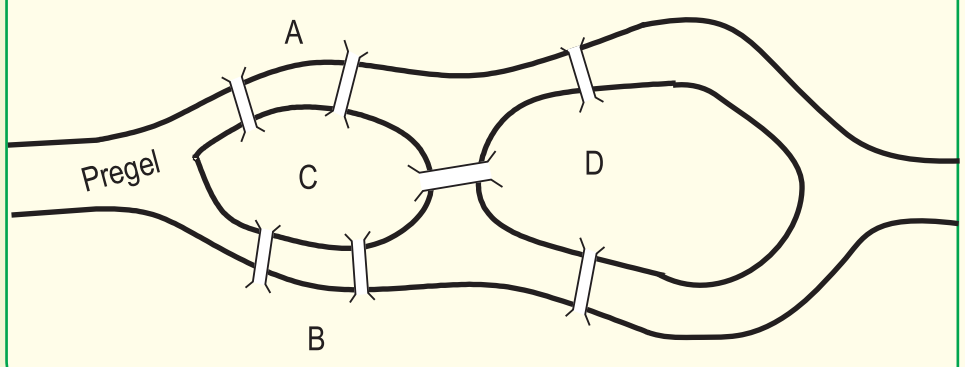
Two graph problems

– are they related?

Problem one: Seven Bridges problem

The river Pregel in Konigsberg city formed two seperate islands (C & D in the figure) connected by seven bridges to the river banks (A & B) and also to each other. Plan a walk so that every bridge is crossed exactly once.



Problem two: Continuous tracing problem

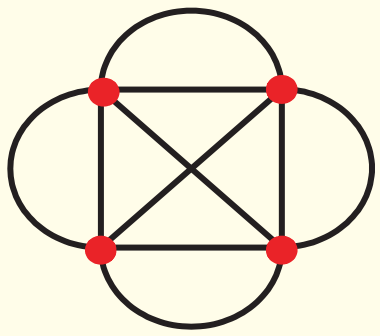


Figure 1

Try to trace Figure 1 in one continous line, i.e., draw the figure without lifting your pen and without retracing the same line.

Are the two problems connected?

The famous mathematician Euler solved the seven bridges problem by realizing that two problems were the same.

To see this draw a graph of the seven bridges problem. (Here ‘Graph’ means not the co-ordinate or X-Y graph, but a network containing points, called vertices, which are joined by lines, called edges.) The seven bridges problem becomes, “Can we trace Figure 2 in one continuous line?”

Graph for the seven bridges problem

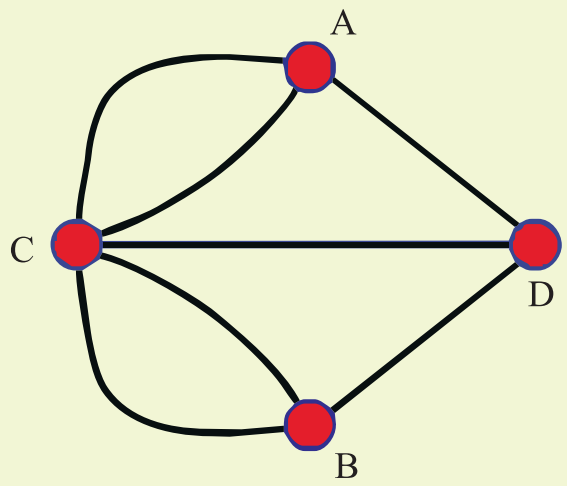


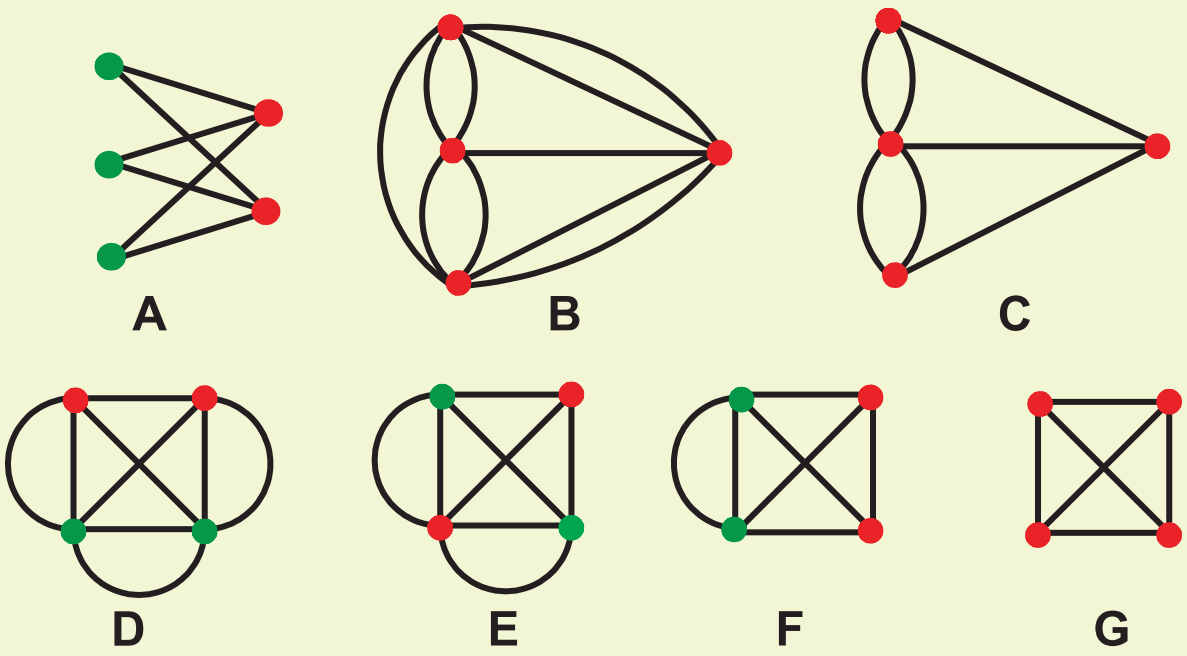
Figure 2

Vertices stand for the two sides of the river and islands. Edges stand for the bridges.

The solution

Spot the odd vertices (odd number of edges attached) and the even vertices (even number of edges attached).

Which of these can be traced in one continuous line?



While tracing figure A in one continuous line, did you notice that an odd vertex has to be a starting or an ending point. Can you think why?

If a figure can be traced in one continuous line it can have at most one starting point and at most one ending point. This means that it can have at most zero or 2 odd vertices.

Now can you see why the graphs in Figures 1 and 2 are impossible to trace in a continuous line?