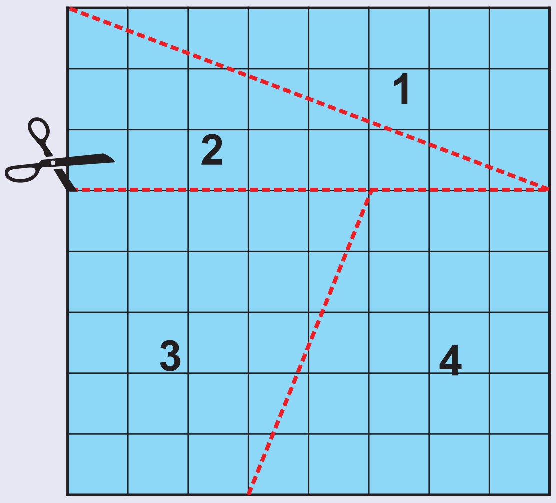
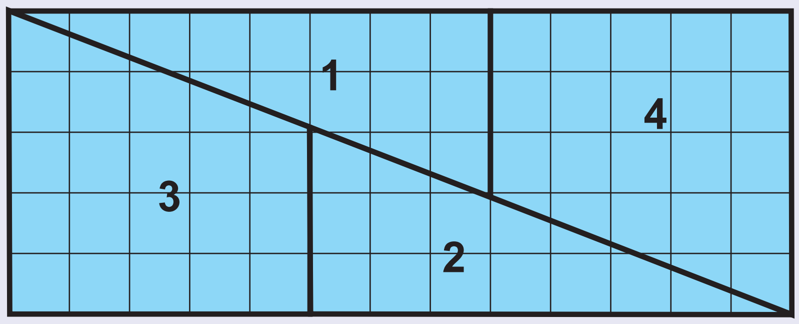
Missing area

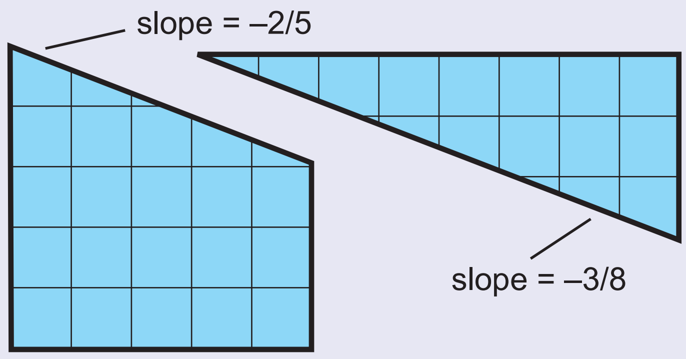
A chessboard has area 8 × 8 = 64 sq units.
Cut it into four pieces as shown.



Reassemble the pieces to form a rectangle.
Now the area is 5 × 13 = 65 sq units !



How did the area increase?
Look carefully the slopes of the edges which meet at the joint.
There is a thin gap where the pieces meet which has an area of exactly 1 sq.unit!
Notice that the sides forming right angles in all the pieces have lengths equal to Fibonacci numbers.



The puzzle exploits three important properties of the Fibonacci numbers.

Fibonacci - Virahanka numbers

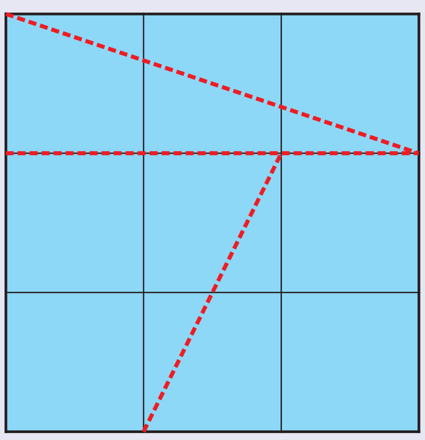
1, 1, 2, 3, 5, 8, 13, 21, ...

Get each Fibonacci number by adding the previous two numbers.

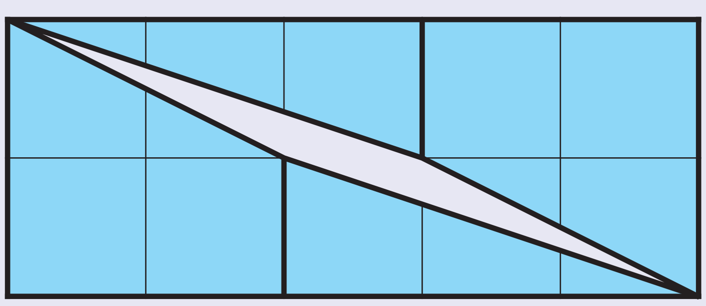
The "Fibonacci" numbers and their properties were known to the Indian mathematician Virahanka in the 7th century CE, nearly 600 years before Fibonacci.

1. Take three consecutive Fibonacci numbers: F_{n-1}, F_n, F_{n+1}.
   Then $(F_{n-1} \times F_{n+1}) - F_n^2 = \pm 1$
   So the missing area in the puzzle is only one unit and is difficult to detect.
2. The ratio F_{n+1} / F_n converges (becomes nearly constant as the numbers get bigger).
   So the slopes at the joint are nearly, but not exactly equal, making the gaps hard to detect.
3. $F_n = F_{n-1} + F_{n-2}$.
   So the square can be dissected and reassembled to form a rectangle.

You can make missing area puzzles from any square whose side is a Fibonacci number greater than or equal to 3.



The missing area puzzle starting with a 3 × 3 square. The gap is very visible.



The missing area puzzle starting with a 5 × 5 square. There is an overlap at the joints instead of a gap because $3 \times 8 - 5^2 = -1$.