Proofs without words!

There are interesting relations between Fibonacci numbers.

Fibonacci - Virahanka numbers

1, 1, 2, 3, 5, 8, 13, 21,...

Get each Fibonacci - Virahanka number by adding the previous two numbers.

Here are two of them:

- $F^2_{n+1} = 4F^2_{n-1} + 4F_{n-1}F_{n-2} + F^2_{n-2}$
- $F^2_{n+1} = 4F_{n-1}F_n + F^2_{n-2}$

Verify them for some Fibonacci numbers.

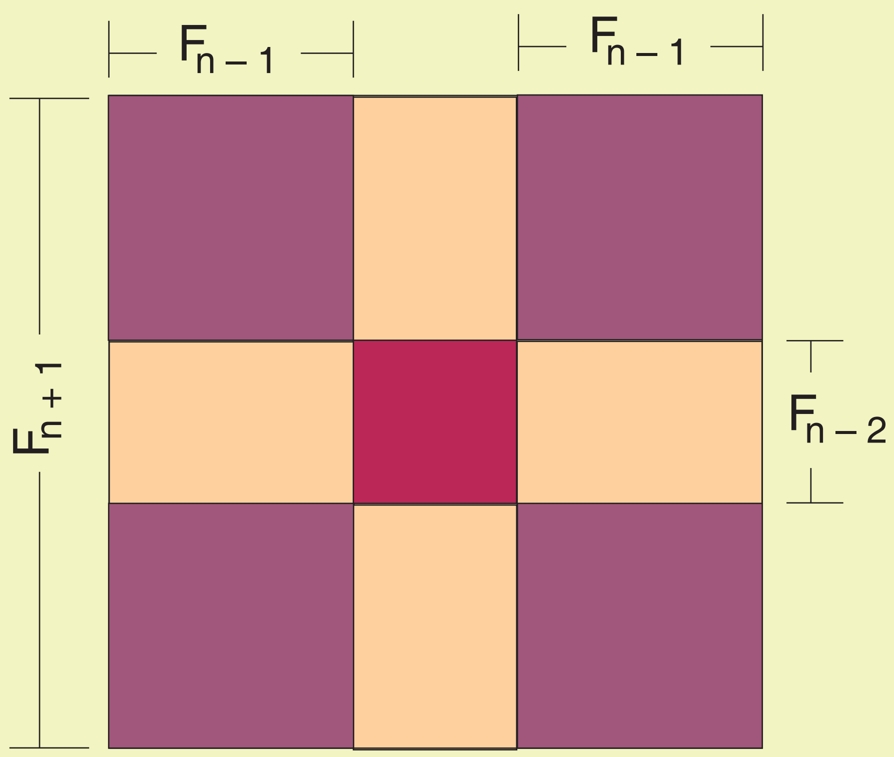
Leonardo Fibonacci

(Leonardo of Pisa) (1120-1245) introduced the use of Indian-Arabic numerals, which included zero, to the West. His treatise, the Liber Abaci (Book of computation) is based on the mathematical works of Arabic scholars.

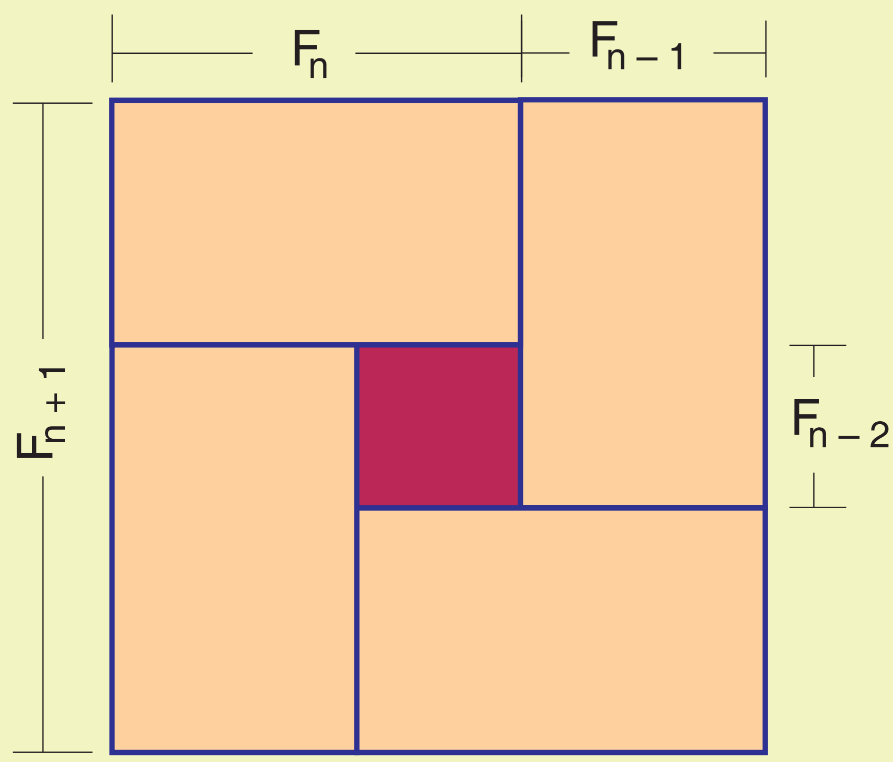
The "Fibonacci" numbers and their properties were known to the Indian mathematician Virahanka in the 7th century CE, nearly 600 years before Fibonacci.

Pictorial proofs of these identities:

$$F^2_{n+1} = 4F^2_{n-1} + 4F_{n-1}F_{n-2} + F^2_{n-2}$$



$$F^2_{n+1} = 4F_{n-1}F_n + F^2_{n-2}$$



Why are Picture 1 and Picture 2 proofs?

Look at the biggest square in Picture 2.

Its area is F^2_{n+1} sq.units.

Now $F_{n+1} = F_n + F_{n-1}$ and $F_n = F_{n-1} + F_{n-2}$

So, the area of the small square is F^2_{n-2} sq.units. Also the area of each of the rectangles is $F_{n-1} \times F_n$ sq.units. So we get,

$$F^2_{n+1} = 4F_{n-1} \times F_n + F^2_{n-2}$$

You can work out and see that Picture 1 is also a proof!