Difficulties with Vectors

Difficulties group	Description	Most frequent incorrect answer for items in this group
Graphical properties	Difficulties in understanding the graphical properties of direction, magnitude, and components of a vector.	Item 5: Two vectors that form different angles with the x axis but pointing to the same region (northeast) have the same direction. Item 4: The y component of a vector has the same value as the magnitude of the vector. Item 9: The x component of a vector has a longer magnitude than the correct one, because it has the same "value" as the magnitude of the vector. Item 2: A unit vector has x and y components of one unit. Item 7: The magnitude of the vector sum of two same-magnitude vectors at 90° is the same as the magnitude of the vectors.
Graphical procedures	Difficulties in understanding the graphical procedures of vector operations: addition, negative scalar multiplication, and cross product.	Item 1: Tip-to-tip error in the addition of two vectors. Item 10: Sketching vector $-2\mathbf{i} + 3\mathbf{j}$ from the tip of the x component to the tip of the y component. Item 11: Vector with incorrect opposite direction in the negative scalar multiplication. Item 19: Add vectors in the subtraction of two vectors in 1D. Item 13: Represent the subtraction vector $(\mathbf{A} - \mathbf{B})$ of $\mathbf{A} = -3\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = -2\mathbf{i} - 2\mathbf{j}$ as $-1\mathbf{i} + 1\mathbf{j}$ (not $-1\mathbf{i} + 5\mathbf{j}$). Item 12: Vector with incorrect opposite direction in the cross product of two vectors.
Geometric calculation procedures	Difficulties with calculations that involve angles, trigonometric functions, and the Pythagorean theorem	Item 17: Calculate the direction of vector $\mathbf{A} = -3\mathbf{i} + 4\mathbf{j}$ as 143.13° (not 126.87°). Item 14: Use the cosine function to calculate the x component of a vector when the angle given is measured from the y axis. Item 6: Calculate the dot product as $AB\sin\theta$. Item 18: Calculate the magnitude of a cross product as $AB\cos\theta$. Item 16: Apply the Pythagorean theorem incorrectly to calculate the magnitude of the vector sum of two vectors at 143.13°. Item 20: Apply the Pythagorean theorem incorrectly when calculating the magnitude of vector $2\mathbf{i} + 2\mathbf{j}$ as 2.
Unit-vector notation calculation procedures	Difficulties with calculations of dot and cross products that involve unit-vector notation.	Item 8: Calculate the dot product $(\mathbf{A}\cdot\mathbf{B})$ of the vectors $\mathbf{A} = 1\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = 5\mathbf{i}$ as $5\mathbf{i} + 3\mathbf{j}$. Item 15: Calculate the cross product $(\mathbf{A} \times \mathbf{B})$ of the vectors $\mathbf{A} = 1\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = 5\mathbf{i}$ as a vector with opposite sign $(15\mathbf{k})$.

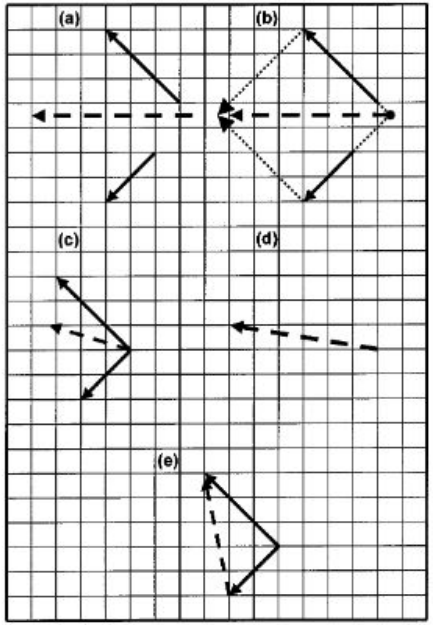

Fig. 3. Common student errors on problem #5 (addition of noncollinear vectors): (a) zero vertical component; (b) split-the-difference algorithm; (c) incorrect parallelogram addition; (d) incorrect horizontal component; (e) tip-to-tip error.

Nguyen and Meltzer (2003)
Barniol and Zavala (2014)

Sources of Difficulties with Vectors

Reflection

bit.ly/tfv-2