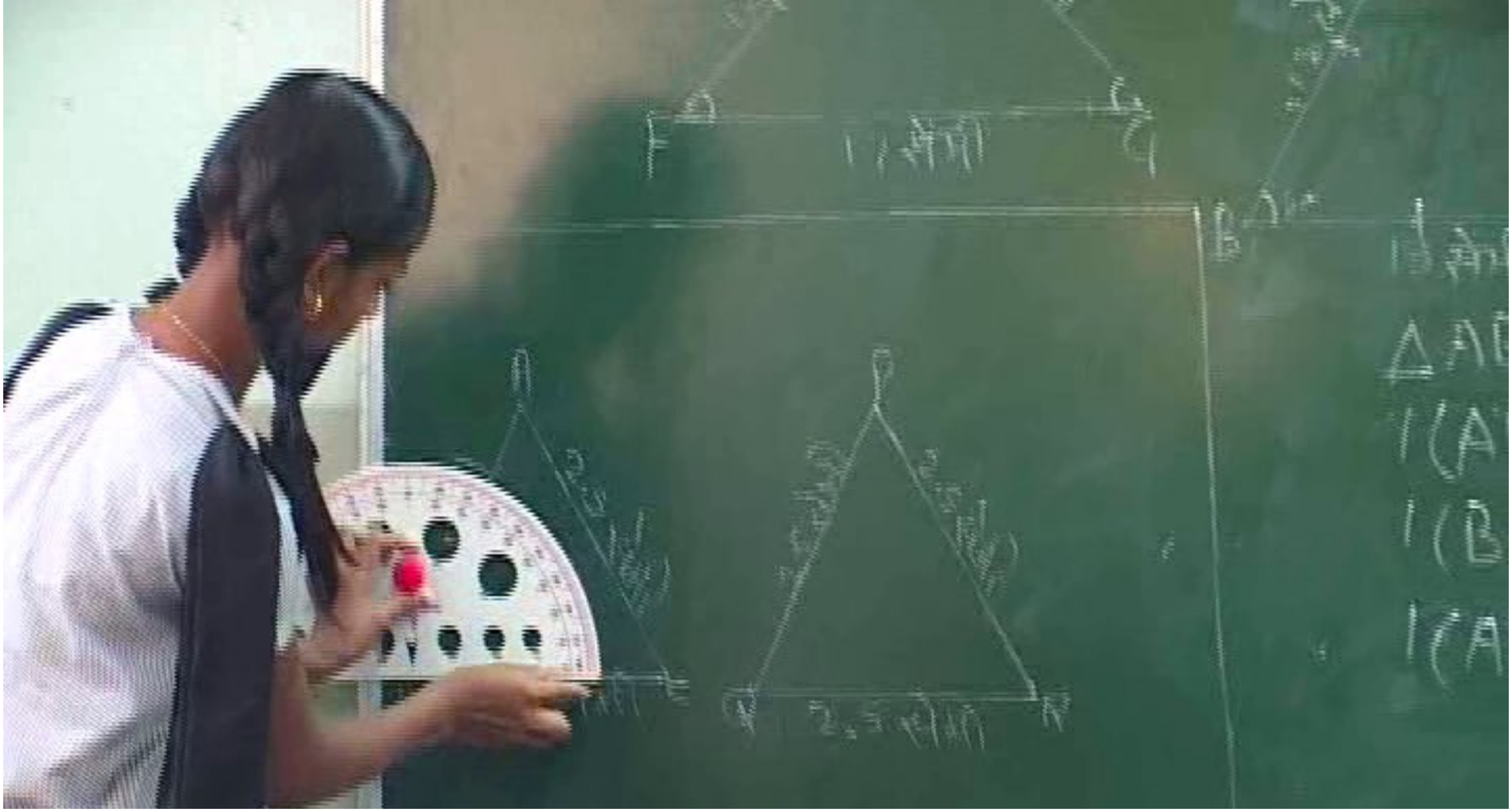
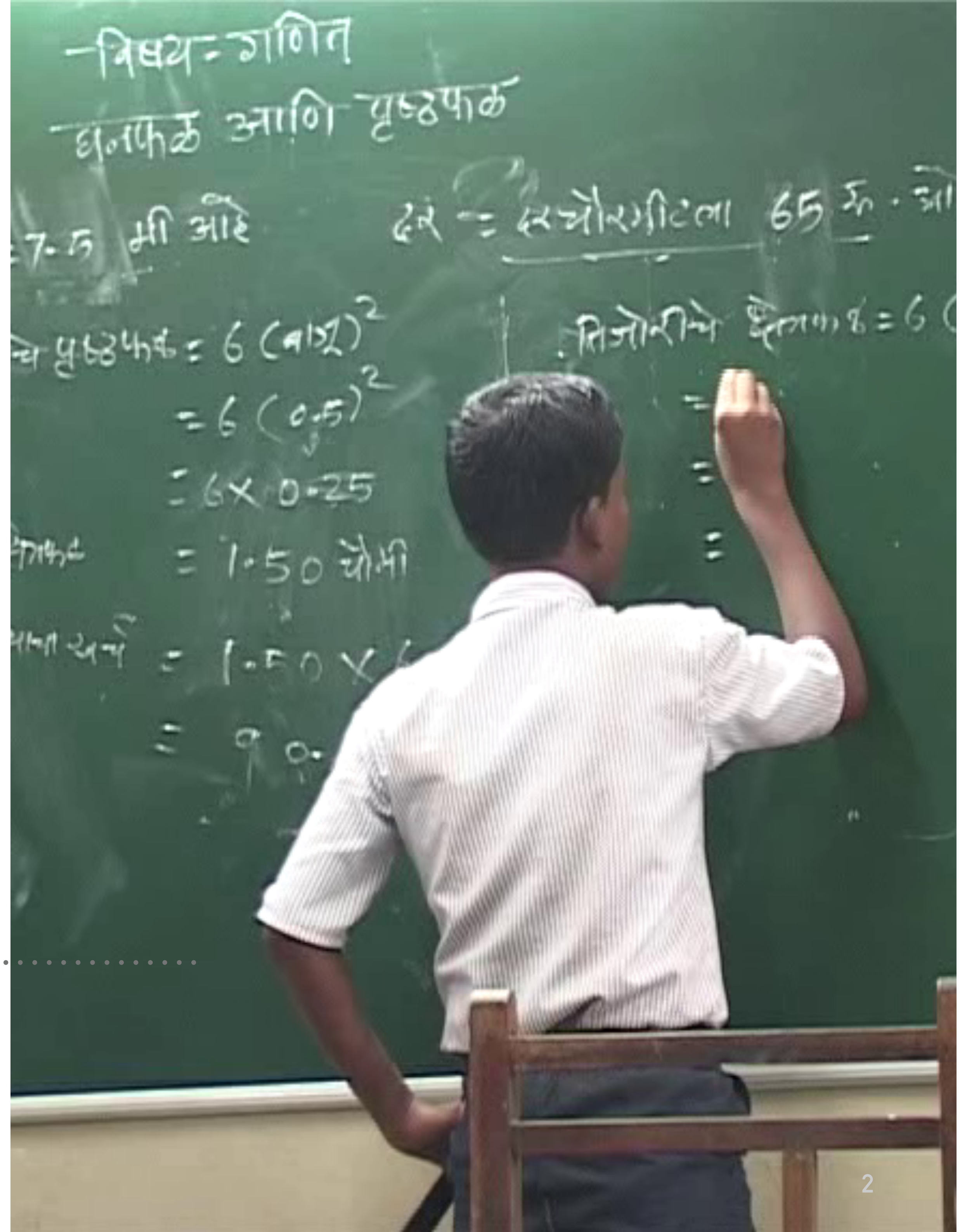
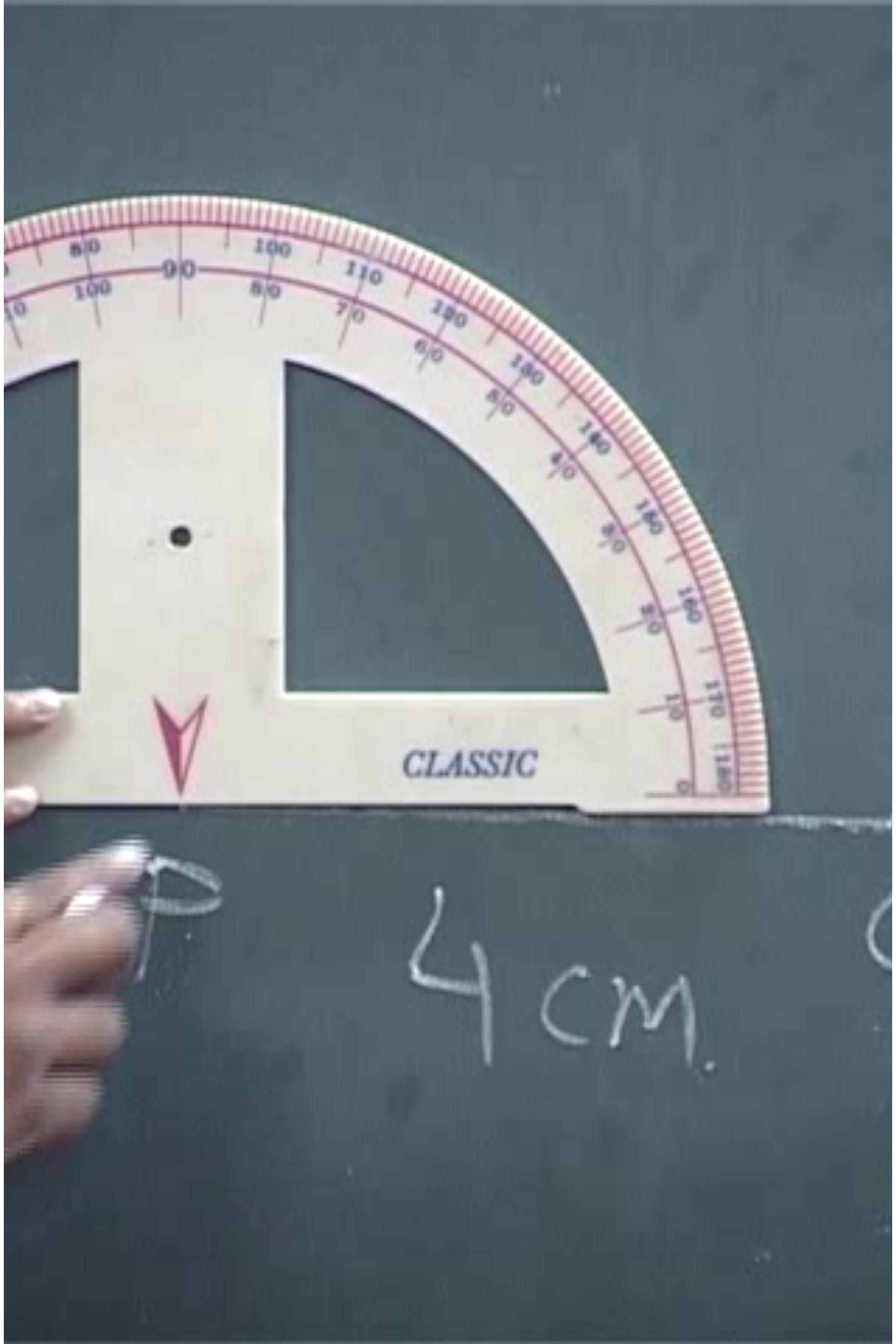


ARTEFACTS OF TEACHING:CASE OF CLASSROOM SCENARIOS

Shweta Shripad Naik
Homi Bhabha Centre for Science Education, TIFR

WHAT IS WORK OF TEACHING MATHEMATICS?



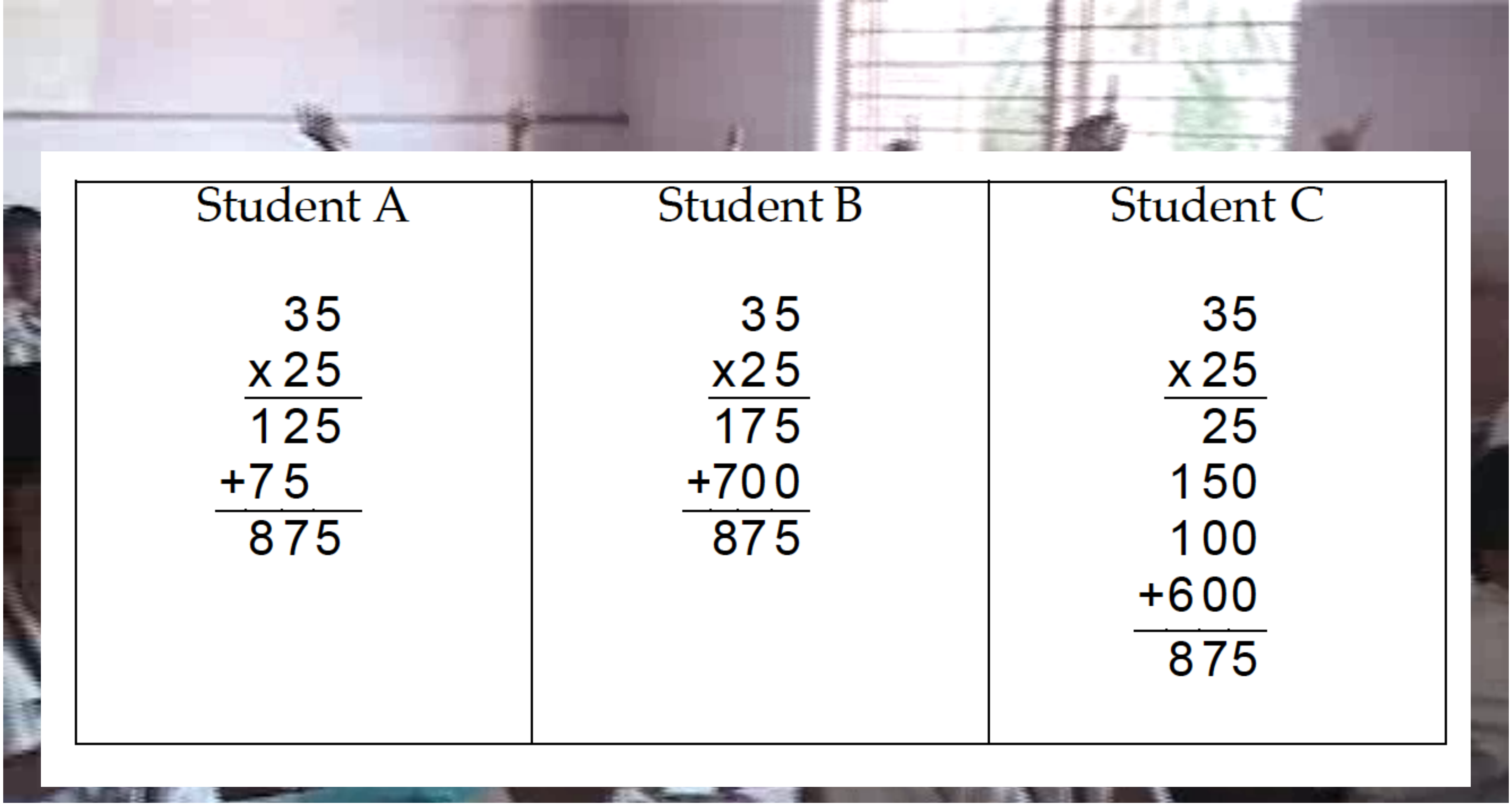


WHAT DO YOU NOTICE?

- When a teacher sees something like this in the class what is work of teaching ?
- Why the protractor has slight extra part on it?

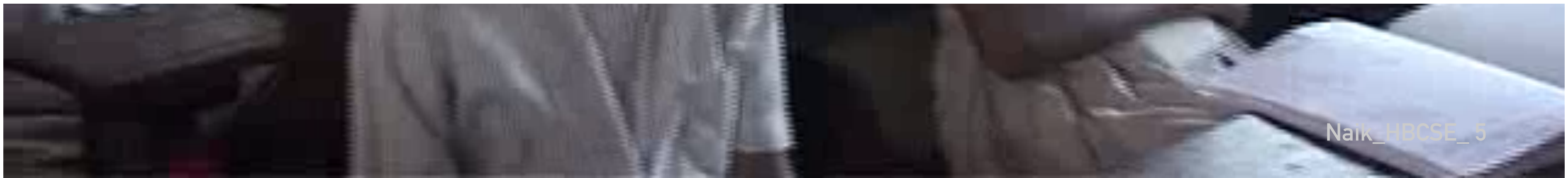
WHAT IS WORK OF TEACHING MATHEMATICS?

(A)	(B)	(C)	(D)
25	25	25	25
X 41	X 41	X 41	X 41
25	25	25	+1000
+100	+ 100	+ 800	25
1025	125	825	1025



Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

Which one of these are generalisable — can be used for any two numbers!



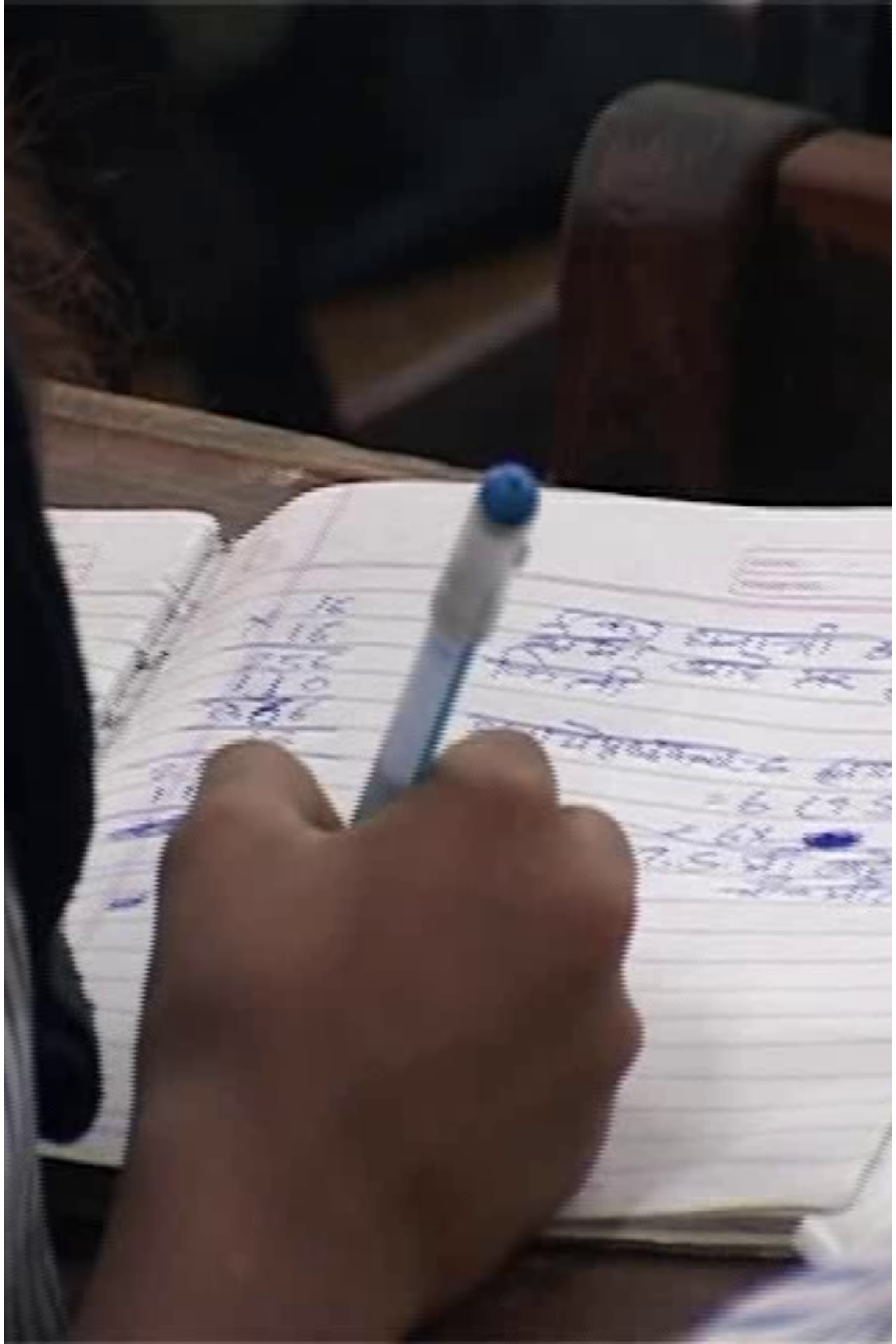
WHAT DOES TEACHING MATHEMATICS INVOLVE?

- Teaching involves deciding where to pay attention and where not to pay attention.

- Mathematics teaching in particular calls for adaptive and responsive style of instruction.

- Assessing on-going instruction requires expert noticing.

Noticing is not an unqualified virtue … key is what to attend to and how to interpret it (D. Ball, p. xxiii)

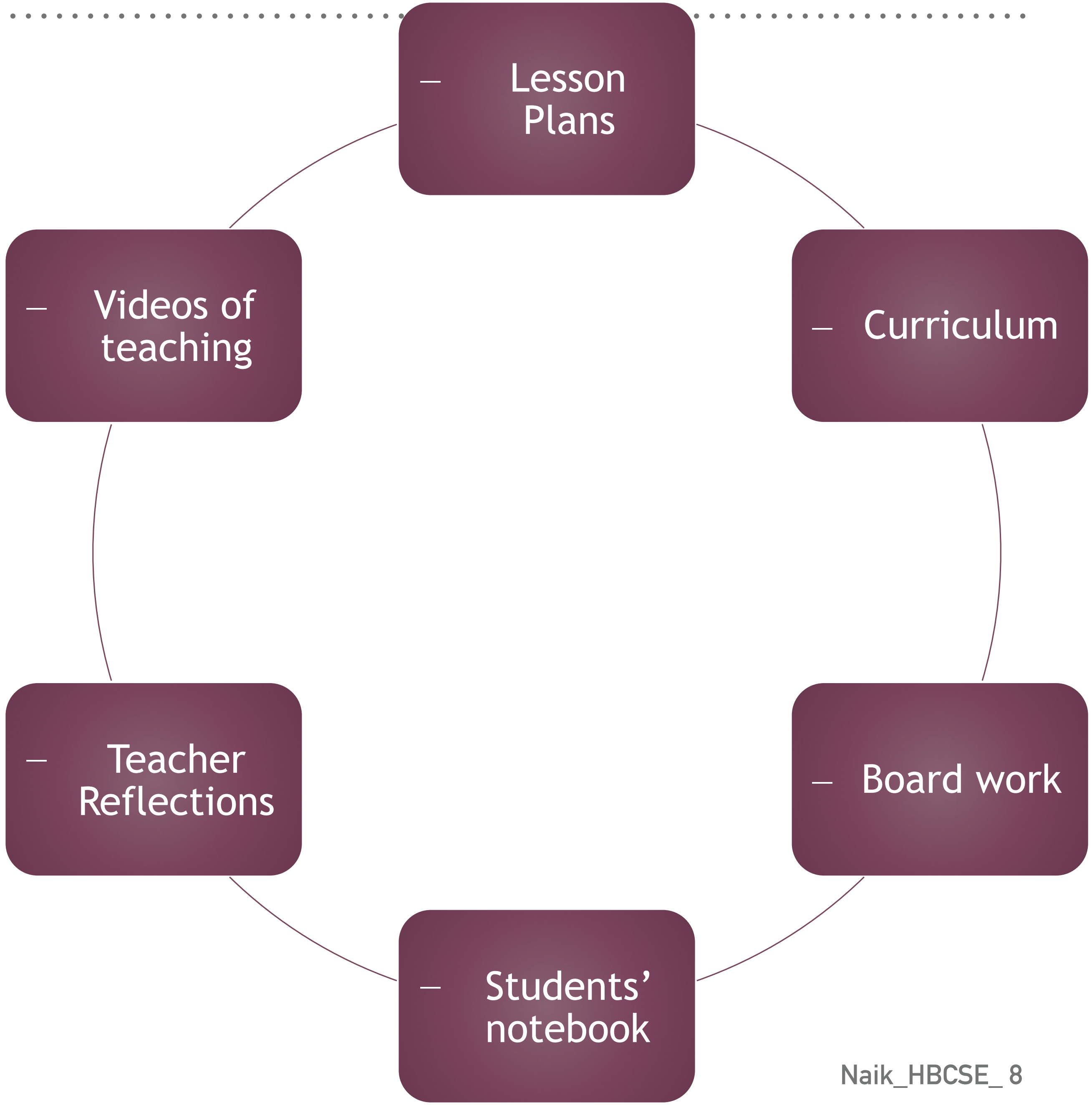


Apart from these general requirement for the teaching mathematics teaching involves

- Choosing examples
- Analysing students’ solutions
- Thinking of representations, contexts and models
- Deciding mathematical explanations
- Making judgements about students’ learning

WHAT ARE ARTEFACTS OF TEACHING MATHEMATICS?

The process of teaching and learning produces artefacts…



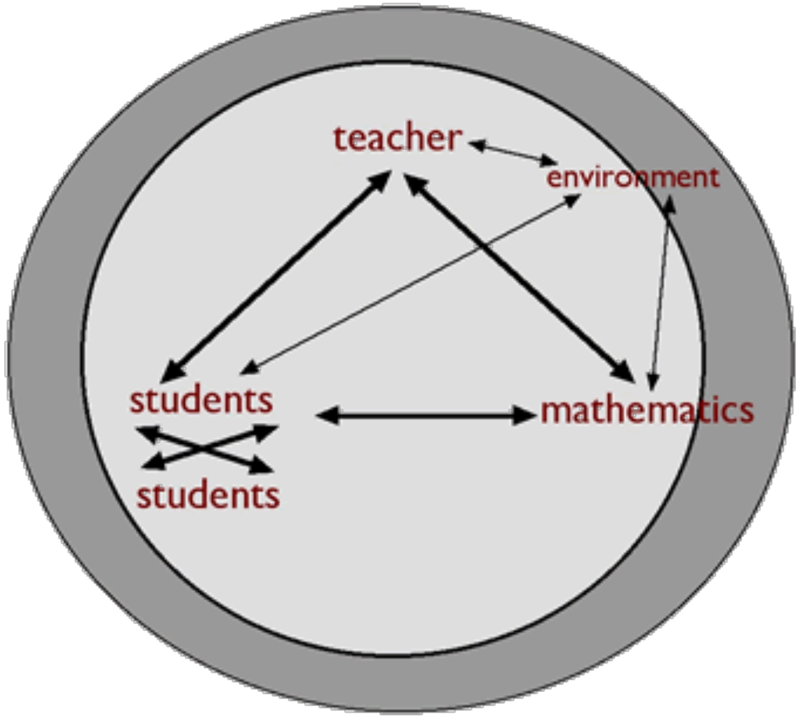
WHY THESE ARTEFACTS?

Requesting or telling students ≠ Teaching students

And similarly

Requesting or telling teachers ≠

Teaching Teachers



ARTEFACTS HELP IMPROVE PROFESSIONAL NOTICING!

- Artefacts are situated in our teaching practice
- Analysing and thinking about artists make us more sensitive about students' thinking, their mathematics and its connection to our instruction

… what teachers attend to as they teach is highly consequential. Given that, the next logical questions become: How and why does it matter, and what can be done about it?

(A. Schoenfeld, p. 224)

AID ANALYSIS OF CLASSROOM SCENARIOS!

- **ATTEND** to the mathematics in the scenario
  - In the question, in student responses, in teachers' instruction
- **INTERPRET** the mathematics
  - What mathematical understanding of students' or teachers' is exhibited in the scenario
- **DECIDE** how to respond
  - Describe some ways you might respond in this scenario and explain the mathematical and cognitive significance why you chose those responses.

SCENARIO 1

In Ragini madam's class students were asked to fill in the appropriate sign < or > or =, in the following comparison of expressions

$59 \div 42 \qquad 359 \div 342$

Shamin said she would write the equal sign because

" $59 \div 42 = 1$ Remainder 17

$359 \div 342 = 1$ Remainder 17

So in both the expressions the answer is 1 and the remainder is 17, and that is why they are equal."

How would you respond to Shamin?

SCENARIO 2

Ms. Shabana's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem.

Student a:
- Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

Student b:
- Check to see whether 371 is divisible by any prime number less than 20.

Student c:
- Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Student d:
- Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

How would you respond to each of the student?

What could be the next course of actions in this class?

SCENARIO 3

Mr. Ajay was reviewing his students' notebooks. While doing so he noticed that one of the students — Gunjana has invented an algorithm that was different from the one taught in the class. Gunjana's work on one of the problems looked like this:

$$\begin{array}{r} 983 \\ \underline{\times \quad\quad 6} \\ 488 \\ \underline{+\,5410} \\ \underline{5898} \end{array}$$

What is Gunjana doing? Is she right?

Can you use her method to calculate 678×9?

How would you respond to her?

SCENARIO 4

Mr. Rodrigues taught his students that a number is called "abundant" if the sum of its proper factors exceeds the number.

For example, 12 is abundant because $1 + 2 + 3 + 4 + 6 > 12$.

He asked students to figure out other numbers that are abundant.

On a homework assignment, many students incorrectly recorded that the numbers 9 and 25 were abundant.

Mr. Rodrigues is thinking about the ways that might have led to such answers. Help him by listing out all possible reasoning that students might have done in this exercise.

And figure out how can he respond to these students.

SCENARIO 5

Leena madam just finished teaching equivalence of expression in her class. She asked her students to explain why the expression $a - (b + c)$ and $a - b - c$ are equivalent.

Here are some responses:

Charu: They're the same because we know that $a - (b + c)$ doesn't equal $a - b + c$, so it must equal $a - b - c$.

Heena: They're equal because of the associative property. We know that $a - (b + c)$ equals $(a - b) - c$ which equals $a - b - c$.

Gurinder: They're equivalent because if you substitute in numbers, like $a = 10$, $b = 2$, and $c = 5$, then you get 3 for both expressions.

Tabu: They're the same because of the distributive property. Multiplying $(b + c)$ by -1 produces $-b - c$.

How would you respond to each student, such that their mathematical understanding is enriched?

SCENARIO 6

> My intentions were to allow students to discover the rules for themselves. The way I did that was to develop a pattern, so I started with $2^5, 2^4, 2^3, \ldots$ and they were supposed to discover a pattern ... students were to see a pattern and to use that pattern to come up with the rules for the zero and negative exponents.

A typical chart for such an exercise might look something like the one below along with expected pupil responses:

Power	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Expected Pupil Response	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

To her surprise, however, Julie found that the patterns the students discovered were not exactly what she had in mind.

> First of all, students had difficulty discovering the pattern to start with. They wanted to say that numbers were decreasing by 2 instead of being divided by 2 as they went down, and so they came up with 2^0 is equal to zero, 2^{-1} is -2, and 2^{-2} is -4, and I'm not sure whether that's because that's the pattern they were seeing or just because people see 2^0 and they want to say it's 0 and 2^{-1} - multiply them together so they get -2. I'm not sure but that's what they came up with.

SCENARIO 7

Majority students in Mrs. Ragini's class said that 3/4 is same as 5/6. They said they are equivalent fractions. For that the students gave the following reasoning.

- In both cases there is one less than the denominator, and therefore if we shade it, one piece would be left
- If you add 2 to 3 you get 5, and if you add 2 to 4 you get 6. There is a common factor here, and therefore they are equivalent.
- If we make the denominators common that would be 6, and numerators too become the same.

How would you respond to these students?

REFLECTING ON WHAT WE CAN DO!

- How can you work with the teachers?
  - Topic wise scenarios?
  - From teachers' own classroom?
  - Use scenarios to enhance content knowledge
  - Use scenarios to make teachers' sensitive to students' thinking and mathematical ideas
  - Use scenarios to make connections with the mathematics in the curriculum

THANK YOU!

For further queries, concerns or suggestions please write to:

shweta@hbcse.tifr.res.in

shwetnaik@gmail.com