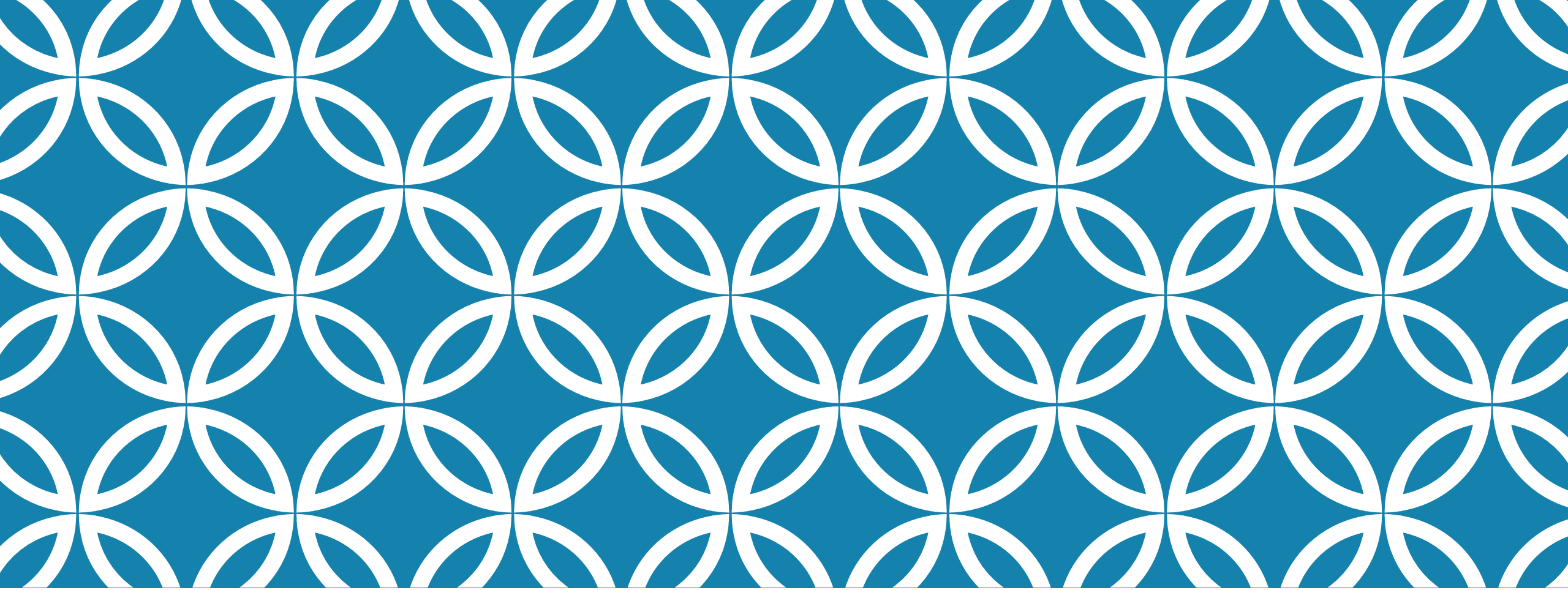

FRACTIONS, RATIOS AND PROPORTIONAL REASONING

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WHAT ARE FRACTIONS?

A student in the class had added fractions like this,

$$\frac{3}{7} + \frac{2}{3} = \frac{5}{10}$$

Why do you think the student added this way?

ARE THESE THE FRACTIONS?

The teacher marked the student's answer as wrong. However, the next day mother of the student arrived in the school with a complaint. He pointed out that the method her child followed was exactly the same that the teacher follows. She gave an example, marks in history $\frac{35}{50}$, marks in Geography $\frac{28}{50}$, are and therefore total marks in Social Sciences are $\frac{63}{100}$

How would you respond to this parent?

FRACTIONS IN EARLY SCHOOLING

A student was asked to fill in the appropriate sign < or > or =, in the following comparison of expressions

$$59 \div 42 \square 359 \div 342$$

A student said he would write the equal sign because

" $59 \div 42 = 1$ Remainder 17

$359 \div 342 = 1$ Remainder 17

So in both the expressions the answer is 1 and the remainder is 17, and that is why they are equal."

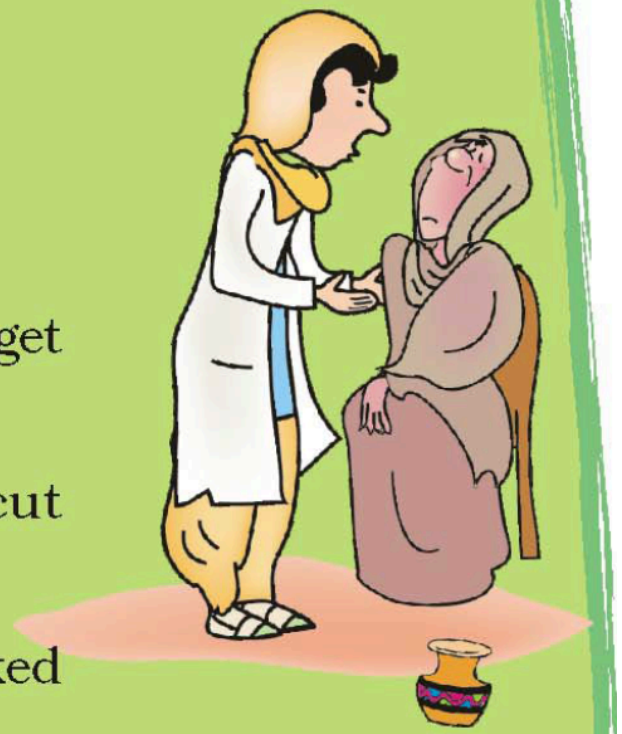
How would you respond to this student?

FRACTION OF A WHOLE

An Old Woman's Will

Once there lived an old woman. She lived with her three daughters. She was quite rich and had 19 camels. One day she fell ill. The daughters called the doctor. The doctor tried his best but could not save the woman. After her death, the daughters read what she had written in her will.

My eldest daughter will get $\frac{1}{2}$ of my camels
My second daughter will get $\frac{1}{4}$ of my camels
My third daughter will get $\frac{1}{5}$ of my camels



The daughters were really puzzled. "How can I get $\frac{1}{2}$ of the 19 camels?" asked the eldest daughter.

"Half of 19 is nine and a half. But we can't cut the camel!" The second daughter said.

"That is right. But what will we do now?" asked the third daughter".

FRACTIONS IN ADVANCED SCHOOLING

Make a story problem that could be used to illustrate

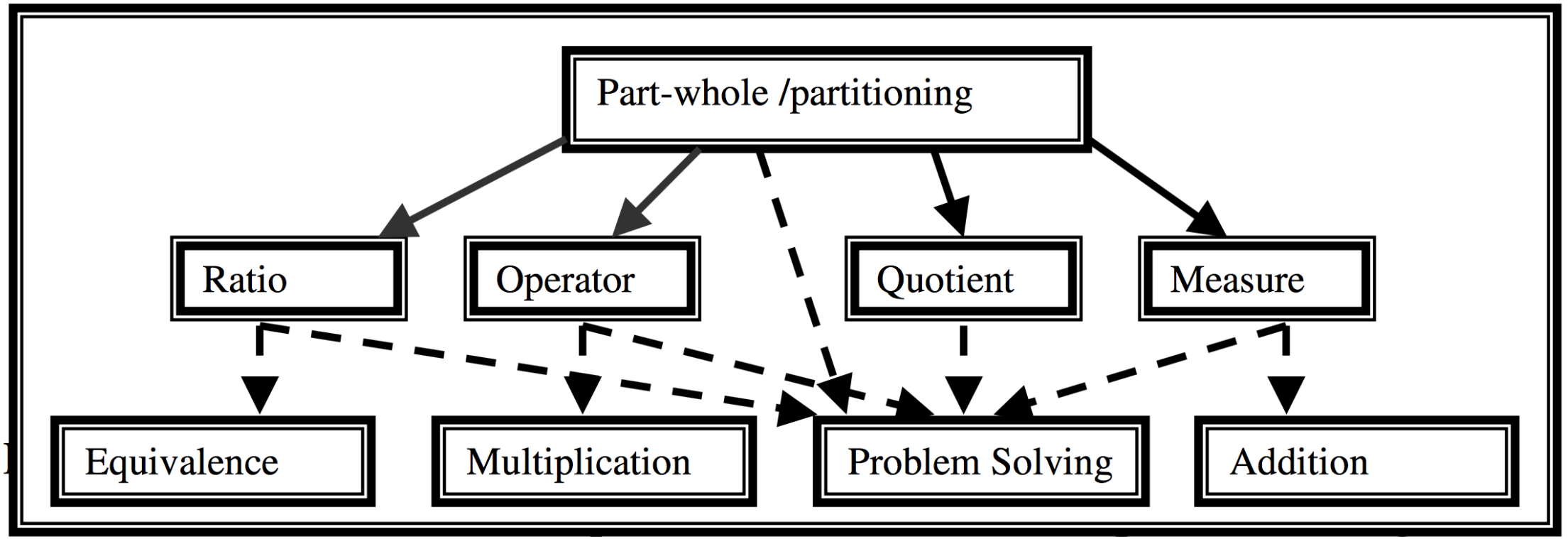
$$1\frac{1}{4} \div \frac{1}{2}$$

RATIONAL NUMBERS

What is denseness in natural numbers? Explain with examples.

How does “denseness” make rational numbers easy or difficult to learn?

FRACTIONS SENSE AND DENOTATION



MEASURE INTERPRETATION OF FRACTIONS NOTATION

Fraction is considered as a number, which conveys the quantitative personality of fractions, how big the fraction is.

A unit fraction is defined (i.e., $\frac{1}{a}$) and used repeatedly to determine a distance (Lamon, 2001; Marshall, 1993)

$\frac{b}{a}$ is $\frac{1}{a}$ used repeatedly b times

SHARE OR QUOTIENT INTERPRETATION OF FRACTIONS NOTATION

Within the quotient sub-construct, any fraction can be seen as the result of a division situation.

In particular, the fraction $\frac{x}{y}$ indicates the numerical value obtained when x is divided by y, where x and y represent whole numbers (Kieren, 1993).

RATIO INTERPRETATION OF FRACTIONS NOTATION

The ratio sub-construct of fractions conveys the notion of a comparison between two quantities; therefore, it is considered as a comparative index, rather than a number (Carraher, 1996).

OPERATOR INTERPRETATION OF FRACTIONS NOTATIONS

In the operator interpretation of fractions, rational numbers are regarded as functions applied to some number, object, or set (Behr et al., 1993; Marshall, 1993).

$\frac{1}{2}$ of Lassi, $\frac{5}{7}$ of cake

PART OF A WHOLE INTERPRETATION OF FRACTIONS

The part-whole sub-construct of fractions is defined as a situation in which a continuous quantity or a set of discrete objects are partitioned into parts of equal size (Lamon, 1999; Marshall, 1993).

From this perspective, the numerator of the fraction must be less than or equal to the denominator.

WHAT SUB-CONSTRUCTS OF FRACTIONS YOU SEE IN OUR TEXTBOOKS?

FRACTIONS IN INDIAN TEXTBOOK

Language used in the textbook (part-part)

Definitions are procedure oriented

- When numerator is less than the denominator the fraction is called as proper fraction.

A paragraph from our textbook

- "The shortest way to find the equivalent fraction in the simplest form is to find HCF of the numerator and denominator, then divide both of them by HCF and get the equivalent fraction." (NCERT textbook)

RESEARCH WORK ON FRACTIONS

Students' difficulties in understanding multiplicative relationships

Understanding fractions through integration of share and measure approach

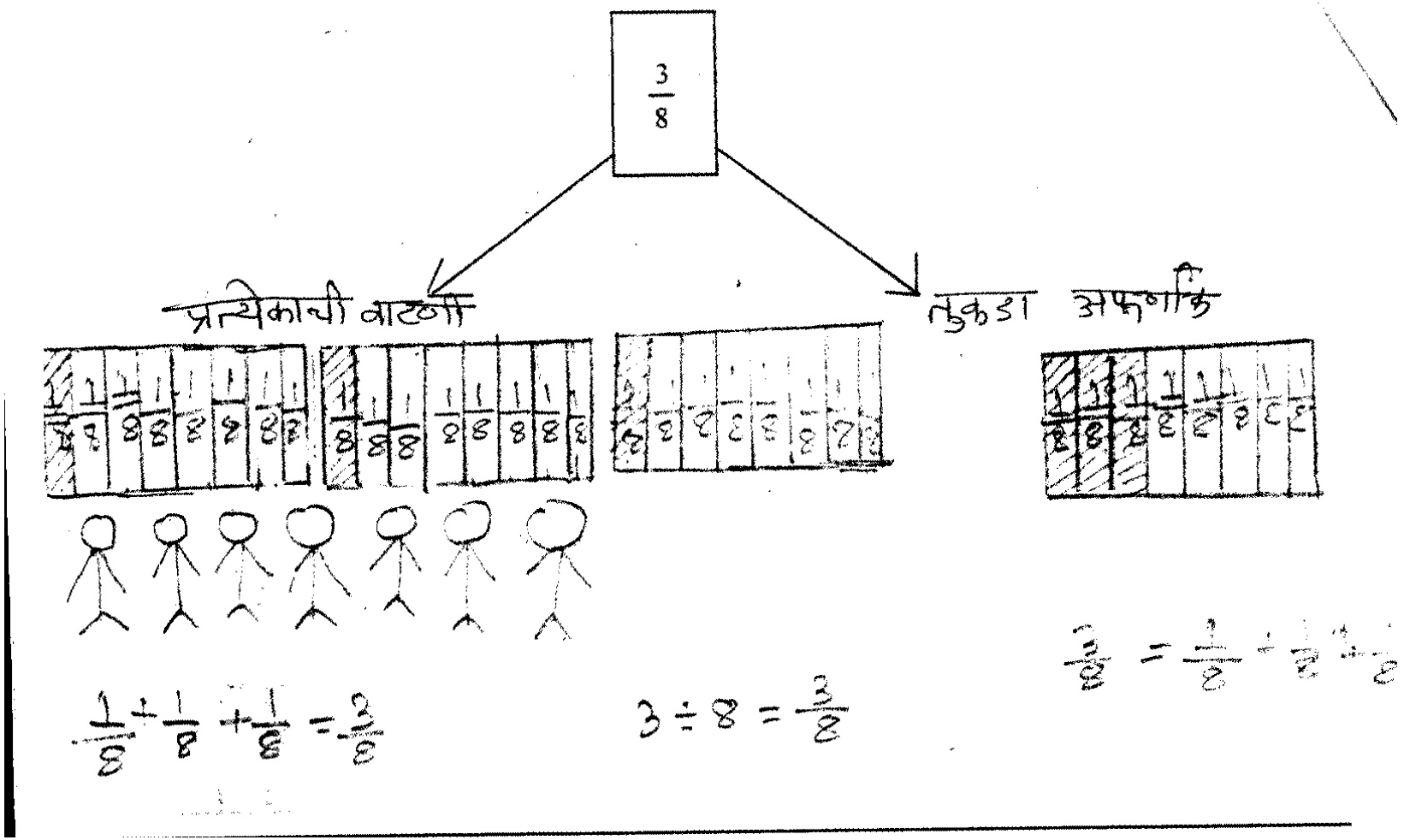
Representation of equivalent fractions by making groups in which everybody gets the same share

Students' representing problems involving fractions on their own/with peers (Naik & Subramaniam (2008))

MULTIPLE APPROACHES TO TEACHING FRACTIONS

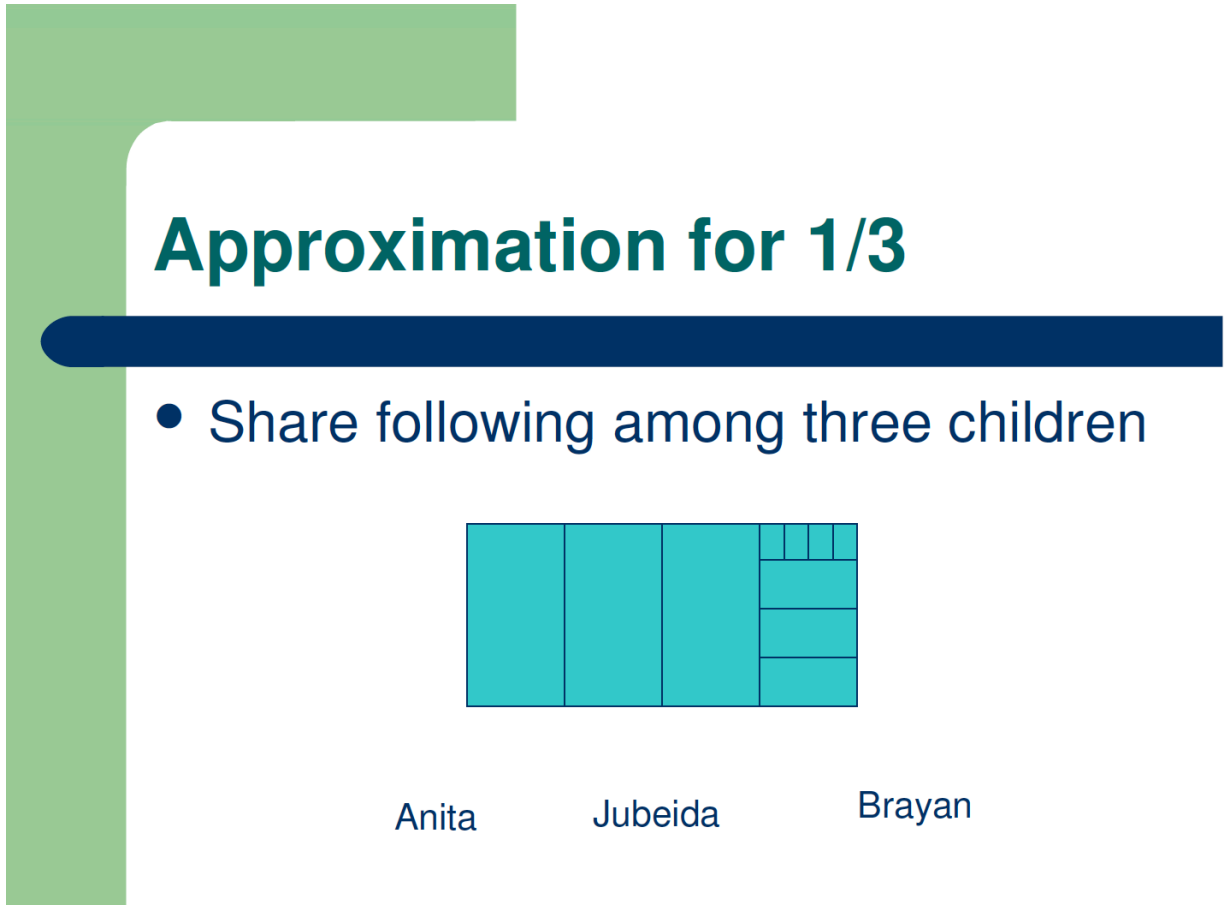
Sharing approaches of learning fractions

Measure approaches of learning fractions



PEDAGOGY FOR SHARING IN FRACTIONS

- In sharing chapatis making different equal parts (e.g. 1/10) is difficult. But reference to a whole as a chapati is easier for students.
- Rectangular whole allows many ways of equipartitioning.
- Sharing tea problems became more interesting, as we worked with three sets of values – total number of cups of tea, number of people to share and money per person.



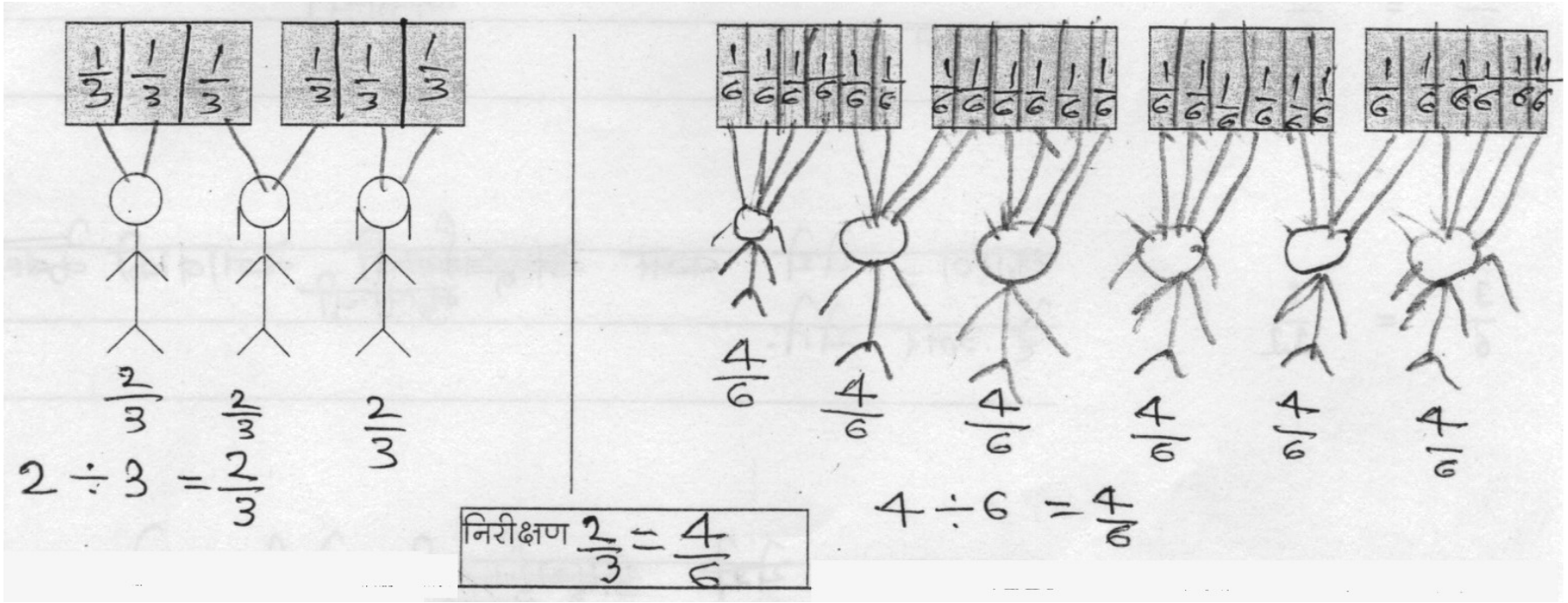
PEDAGOGY FOR MEASURING IN FRACTIONS

Various informal units rediscovered in students' measurement

– Half of half which they understood as a quarter

– Half of a quarter / quarter of a half

– Quarter of a quarter



STUDENTS REASONING IN THESE TWO APPROACHES

In comparison tasks students drew on both the measure and the share interpretations.

Fractions with the same numerator:

- As the number of cakes to share are same, the group where more number of children are there will have a smaller share.
- As the number of pieces are same what matters is the size of the unit.

Comparing fractions with half:

- Students reasoned that the fraction is equal to half when the number of cakes is exactly half the number of children or when the number of pieces taken is exactly half the total number of pieces.

STUDENTS REASONING IN THESE TWO APPROACHES

In comparing the fractions $\frac{4}{5}$ and $\frac{6}{7}$,

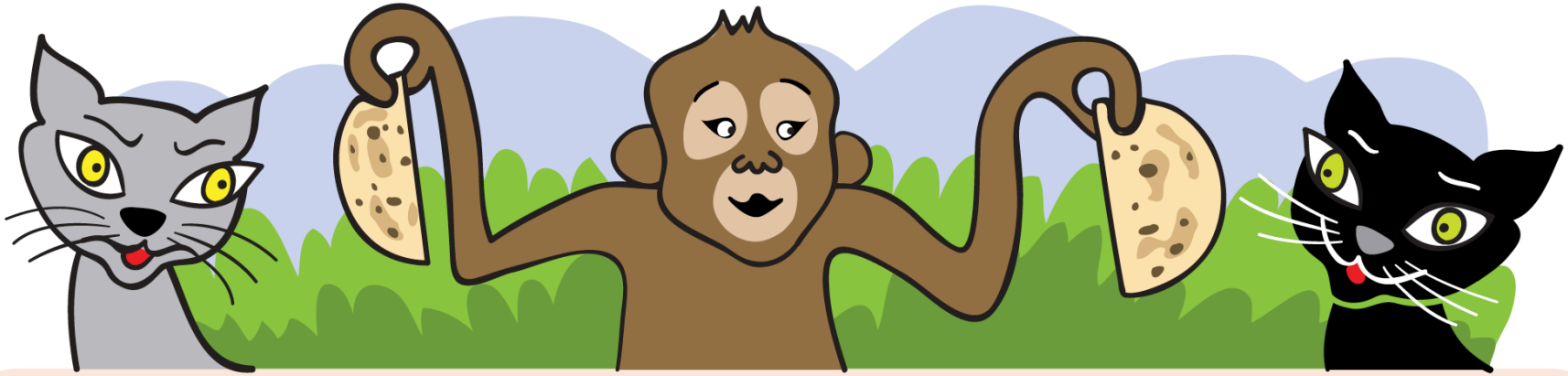
- Even though both the fractions $\frac{4}{5}$ and $\frac{6}{7}$ need one more piece to complete a whole. $\frac{4}{5}$ needs one piece of $\frac{1}{5}$ and $\frac{6}{7}$ needs one piece of $\frac{1}{7}$. but $\frac{1}{5}$ is more than $\frac{1}{7}$ as one cake is shared among 5 children only, hence $\frac{4}{5}$ is away from the whole.

Improper fractions were understood as

- As number of cakes are more than the number of children to share. Obviously each child will get at least one cake

NCERT TEXTBOOKS

Mintu cat and Mottu cat were friends. Once they stole a chapati from Malini's kitchen. I will take it — said Mintu. No, I will take it — said Mottu. While they were quarrelling, there came Tittu Monkey. Hi! What is the problem? why are you quarrelling? — he asked. "We don't know how to divide this chapati between us — the cats said. OK! don't worry. I will divide the chapati equally for both of you — he said. Clever Tittu divided the chapati like this:



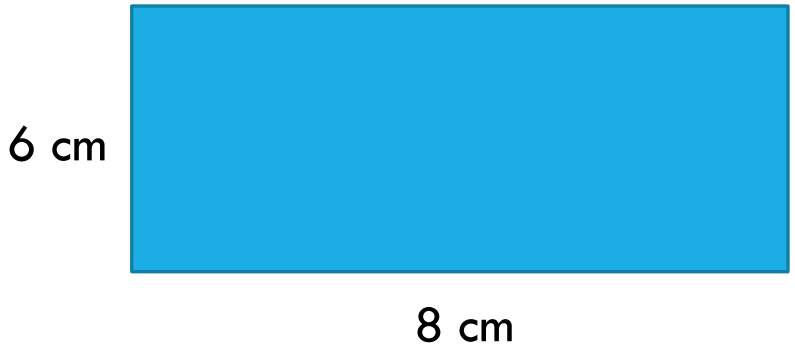
These are not equal, the left part is bigger — Mintu and Mottu said. Oh, no problem, I will make it equal — Tittu said. He then cut a part of the left piece and ate it.

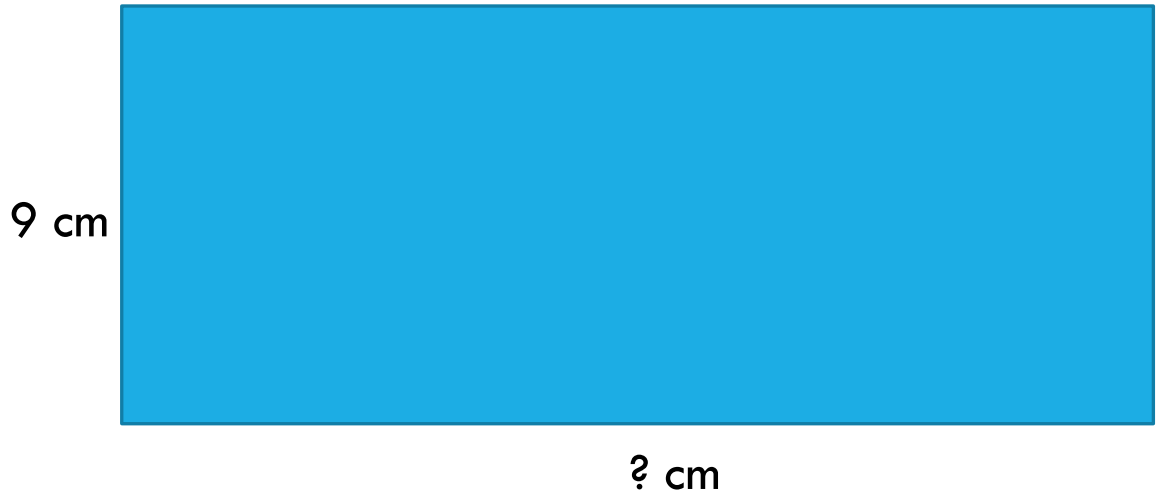
DESIGN SITUATIONS

Additive reasoning

Multiplicative reasoning

MULTIPLICATIVE RELATIONSHIPS

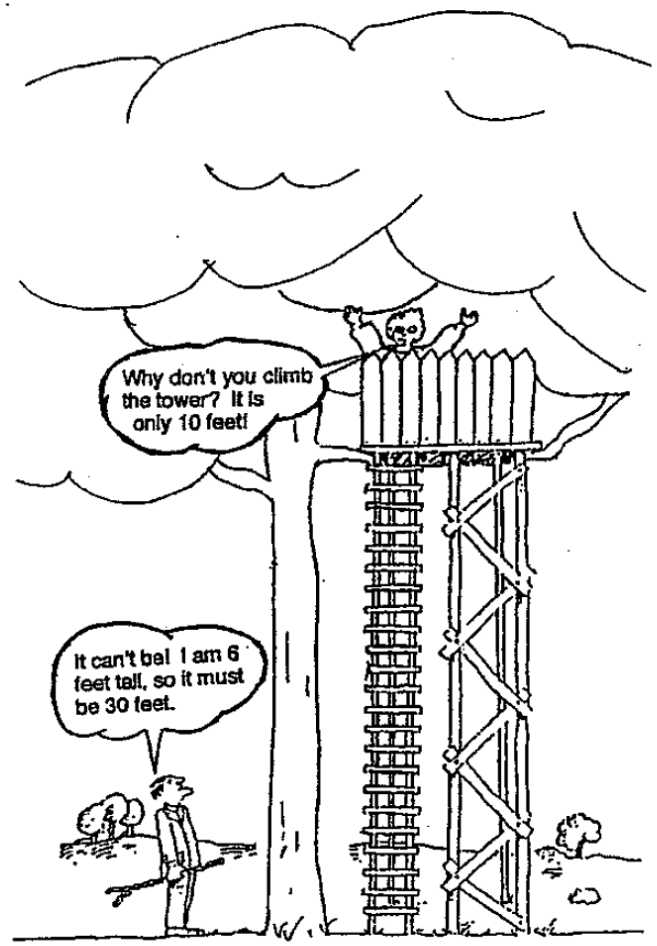




STUDENTS' MULTIPLICATIVE THINKING

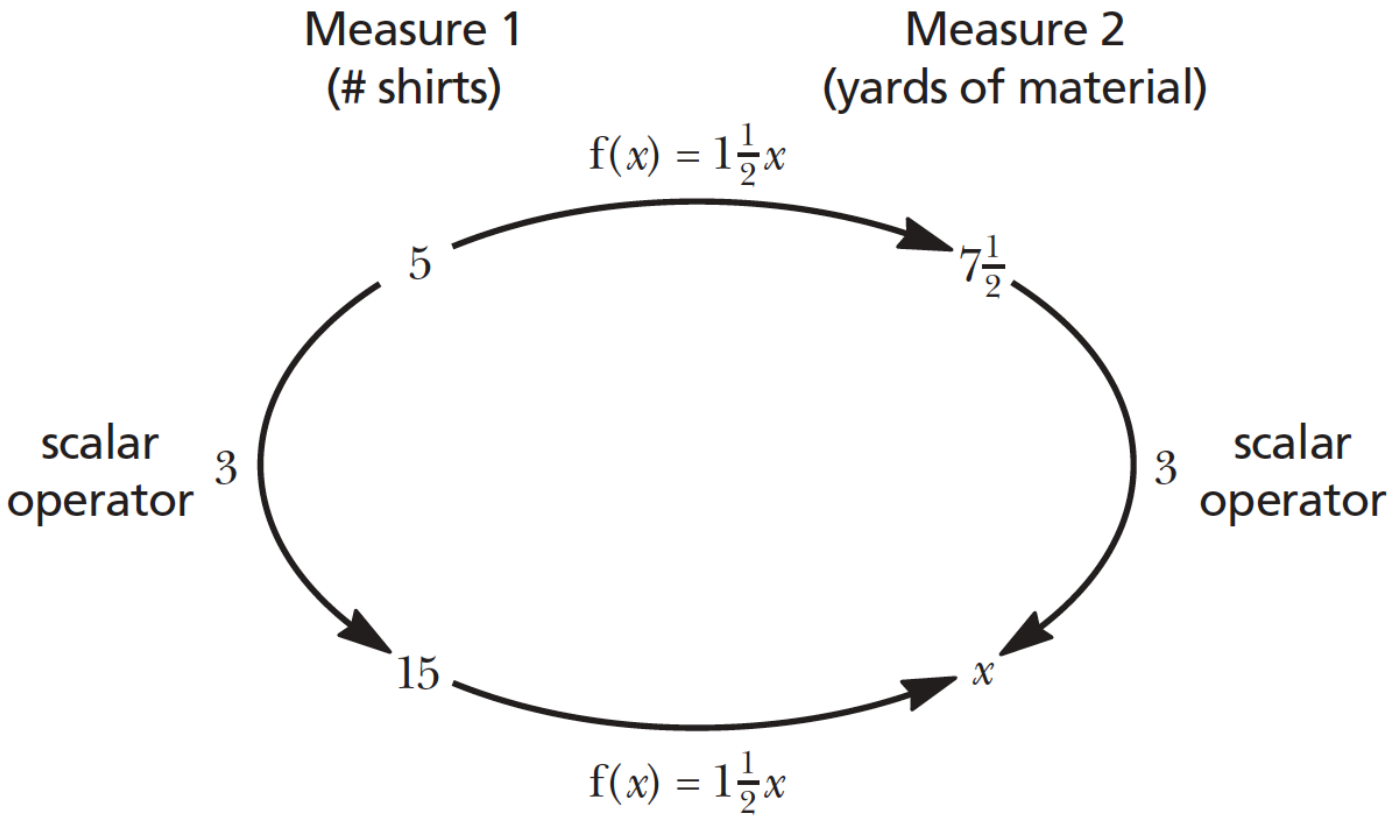
Student Strategies: Grade 6

Some middle school children discussed the tree house problem and the responses from students A, B, C, D, and E are given here. First, solve the problem yourself and explain your reasoning to someone else, then rank the student responses according to the sophistication of their mathematical reasoning.



These people disagree on the height of the tree house. How high do you think it is? Explain your reasoning.

FRACTIONS AND MULTIPLICATIVE THINKING



If Neeraj can sew 5 team shirts with $7\frac{1}{2}$ yards of material, how many yards material he will need for 15 team shirts?

PROPORTIONAL REASONING

Indicator of students understanding of rational numbers and multiplicative concepts

Foundation of more complex ideas

What is proportional reasoning?

What is proportional reasoning according to Lamon?

PROPORTIONAL REASONING

Proportional reasoning refers to the ability to scale up and down in appropriate situation and to supply justifications for assertions made about relationships in situations involving simple direct proportions and inverse proportions (Lamon, 1999)

UNITIZING: BRIDGE BETWEEN FRACTIONS AND PROPORTIONAL REASONING

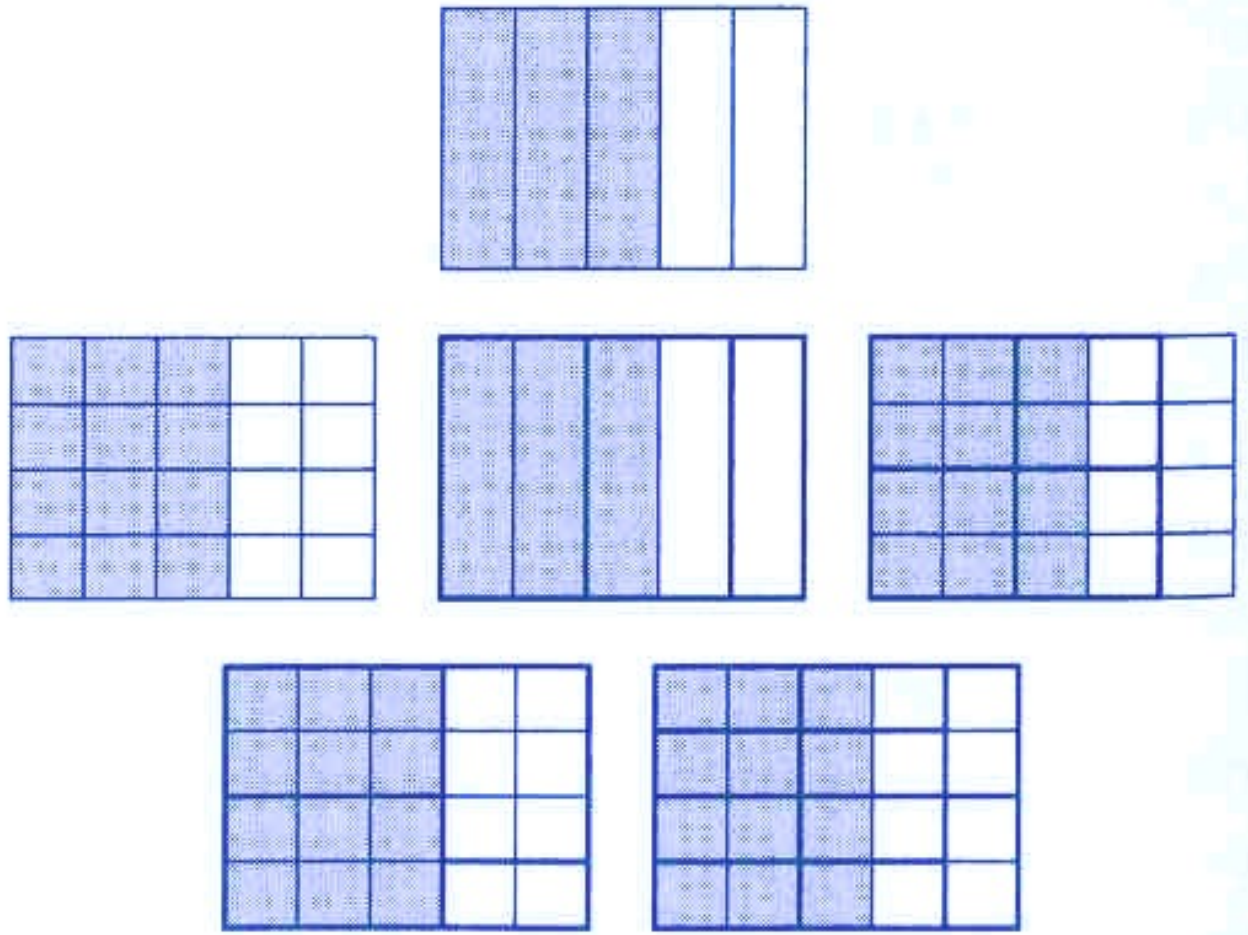


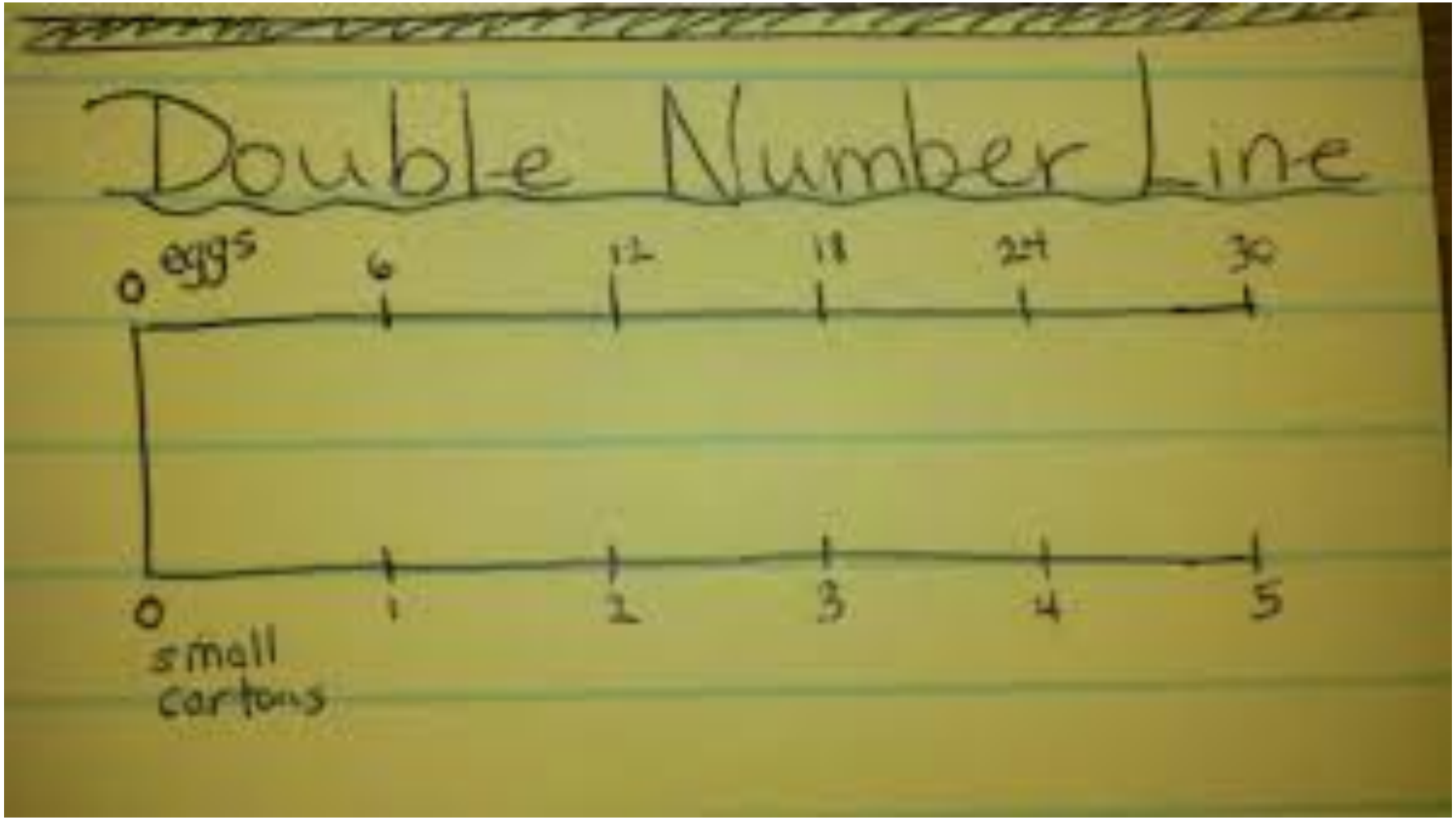
Fig. 9.2. Chunking an area into different-sized pieces

DIRECT AND INVERSE PROPORTIONS

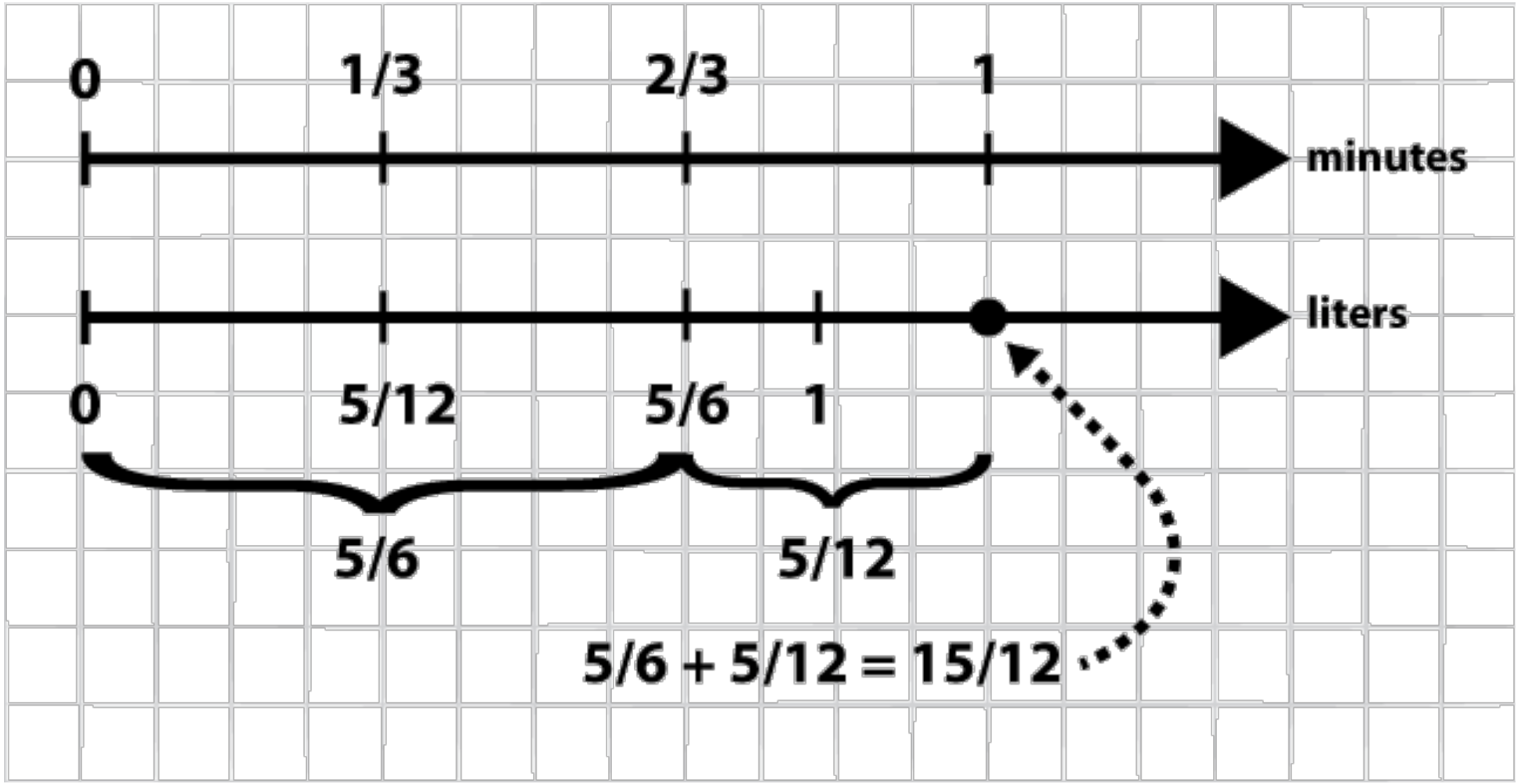
Make two situations one that involves a direct proportion and other that involves inverse proportion.

Can you make representation for the two?

DOUBLE NUMBER LINE



DOUBLE NUMBER LINE



HOMEWORK

Design a representation for a situation that involves inverse proportion

CONTACT DETAILS

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