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CHANGING TEACHER KNOWLEDGE-IN-PRACTICE: THE CASE OF DECIMAL FRACTIONS

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In this paper, we examine elements of knowledge implicated in the classroom teaching of decimal numbers by a middle grades mathematics teacher, Nandini. The study is based on an analysis of "paired episodes", i.e., episodes of classroom teaching of the same topic by the same teacher over two consecutive years. In the episode from the second year of teaching, Nandini's is more responsive to students, and draws a richer store of specialized content knowledge and knowledge of content and students in her responses. We explicate these knowledge elements in our analysis. The pattern of enhanced responsiveness and richer knowledge elements in play is consistent with comparisons of other paired episodes.

BACKGROUND

The study reported in this paper is a part of a larger project investigating the links between mathematics teachers' knowledge, classroom practice and sensitivity to students' thinking. Recent literature argues for practice-based approaches to characterize specialized knowledge of mathematics teachers required for effective teaching (Bass & Ball, 2004; Mitchell, Charalambous & Hill, 2014). In this paper, we examine elements of knowledge implicated in a teacher's practice while teaching a specific mathematical topic as it changed over successive years in terms of responsiveness to student thinking. The study is based on an analysis of "paired episodes", i.e., episodes of classroom teaching of the same topic by the same teacher over two consecutive years.

Several researchers have examined the relation between teachers' content knowledge and its implications for teaching practice. In their framework on Mathematical knowledge for teaching; Ball, Thames and Phelps (2008) identify components of such knowledge, of which specialised content knowledge (SCK) and knowledge of content and students (KCS) are topic-specific in nature. SCK enables teachers to engage in teaching tasks such as linking representations, responding to students' why questions, explaining mathematical goals and giving or evaluating explanations. KCS includes knowledge of students' difficulties in relation to specific topics and is a part of teachers' pedagogical content knowledge. Rowland (2009) proposes a framework to analyse pre-service teachers' knowledge-in-use by connecting knowledge with kinds of teacher actions. The categories of teacher knowledge in this framework are: *foundational* propositional knowledge and beliefs learnt from training or courses, the teacher's ability to *transform* content knowledge through the choice of powerful pedagogical forms, c*onnecting* lessons and sequences of lessons to present mathematical ideas more coherently with a structural focus, and readiness to respond to unanticipated *contingencies* arising from students' engagement with the concepts.

Our analytical framework draws on both these frameworks. The knowledge quartet framework serves as a guide in relating teacher actions to knowledge elements. We use the categories of SCK and KCS to call attention to fine-grained aspects of topic-specific knowledge implicated in teaching practice and to identify the distinctions as well as the links between SCK and KCS. We believe that the characterization of knowledge discerned from teachers' practice will add to the repertoire of topic-specific knowledge components identified in the literature on teacher knowledge.

RESEARCH STUDY

The larger research project, which aimed at enhancing teacher knowledge and responsiveness through engaging in classroom based tasks, involved working with four school mathematics teachers. All the teachers had a bachelor's degree in mathematics or physics and a one-year degree in education and had more than 20 years of experience in teaching mathematics. Two teachers taught the primary grades (Classes I-V, Years 6-11) and two teachers taught the middle grades (Classes VI-X, Years 11-15) in a school in Mumbai, which caters to children from a mixed socio-economic background. Data was collected for two academic years (2011-13). In this paper, we discuss the case of Nandini (pseudonym), a middle school mathematics teacher. Nandini had been teaching mathematics and physics to the middle grades for over 21 years. The teaching episodes and interactions used in this paper are from Nandini's teaching of decimal fractions. We choose her case since the changes in her teaching practice in the second year were significant and evident.

The study follows a case-study methodology with a intermix of exploratory and interventionist components. Data was collected in the form of classroom observations, task-based interviews, semi-structured interviews, and teacher researcher meetings. These meetings were the core of the intervention component in the form of discussions centered around classroom tasks and possible or actual student responses. For the purpose of this paper, we use the data from classroom observation notes, audio and video records, field notes and audio-records of discussions with Nandini prior to and after the lesson observed. Descriptive codes were developed by studying the transcripts of classroom observations. Data analysed for this paper include paired episodes from Nandini's teaching of decimal fractions in two years in Class VI. The episodes are paired on the basis of the mathematical idea being discussed in the classroom. Through a comparison of episodes, we attempt to characterise the change in Nandini's teaching practice and describe the knowledge implicated in her teaching actions and decisions.

ANALYSIS & FINDINGS

The data from Nandini's classroom teaching and interactions with the researcher revealed change in her practice and therefore knowledge of teaching decimal fractions. We discuss an episode from Nandini's teaching in each year to exemplify the nature of change and identify considerations guiding the change. The following episode focuses on the conversion between measurement units, centimeter and millimeter.

Episode 1: Conversion of measurement units

The ruler was used in the first lesson on decimal fractions in Year 2012 to measure the length of the duster in the classroom. Since the length was between 17 and 18 centimeters, the need for a numerical measure of this length was used to introduce decimal numbers. The episode for discussion is from Day 5 of decimal teaching. The class was engaged in solving the textbook problem of conversion from millimeters to centimeters. The teacher initiated the discussions by drawing students attention to the divisions between 0 and 1. Teachers cued students in counting every division and then named the indicated units as "millimeters". She then defined 10 millimter divisions as equal to 1 centimeter (The episode transcript will be included in the full paper). Nandini introduced the decimal representation for the sub-units that make a centimeter and named it as 0.1, 0.2, and so on. The pattern of 0.1, 0.2, 0.3... was extended to convert bigger lengths from millimeter to centimeter. In the remaining lesson, the class conversion lengths like 30mm, 16mm, 4cm, 2mm, etc., to centimeter units. Nandini went on in subsequent lesssons to introduce the hundredths place value, an area representation of a 10 by10 grid, without any reference to the measurement of length or the number line.

In the teaching of decimal fractions, measurement is chosen as a context for understanding the relation between different units. The multiplicative relation between units is structurally the same as the relations that occur in the decimal representation of numbers involving powers of ten. Apart from being a context for decimal representation, length measurement offers a linear representation from a ruler to a number line. This relationship between the measurement context and the number line representation is used by Nandini, to introduce the conversion from the smaller to the bigger unit. When doing conversion, measures of length were identified in both units, for e.g., 1 millimeter and 0.1 centimeter. However, neither the fraction representation of the relation between units nor the addition of fractions, which were known to the students, were mentioned. The use of the fraction equivalent of the measure might have helped students in connecting the decimal-fraction representation and justified the link between the two units. 1 millimeter is 0.1 centimeter because of the relation between the number of sub-units that constitute the bigger unit. Also, the relation between 1 millimeter, one-tenth of a centimeter and 0.1 centimeter might have been strengthened by discussing the relation between place values, which in turn, could have been represented using the fraction notation. The structural similarity of the relation between the units of the metric system and decimal representation is an important part of teacher's knowledge in this case. Another important piece of knowledge is the affordance of a representation used. Nandini used the number line representation to introduce the relation of tenths, while hundredths were introduced using a grid. Thus, there was a lack of consistency in the representations used for extending the place value from tenths to hundredths. The choice of different representations for tenths and hundredths created a disconnect between the continuity of units among students. This was evident in their difficulty in using a number line representation to show a decimal number with hundredths place value.

Episode 2: “One division after one”

This episode is from Day 7 of decimal teaching in the Year 2013. In this lesson, Nandini drew a ruler on the board explaining the purpose of scaling up the divisions for visibility. A student measured a duster using the ruler drawn on the board and said '*one division after one [centimeter]'*. Nandini revoiced the student's utterance and asked the whole class to think about this measure. She encouraged students to represent the measure in different units. Students’ responses included: 11 millimeters, 1 centimeter 1 millimeter, 1 and one-tenth centimeter. Nandini asked students to justify their responses. The lesson concluded with the consensus that 1.1 centimeter is the same as 1 centimeter 1 millimeter, 1 centimeter and one-tenth of a centimeter, and 11 millimeters. Students used the ruler, the place value of 1 in different positions, and the relation of one-tenths as justification to reach this conclusion. Further, in the same lesson, the students used the decomposition of a decimal number, *'2 millimeter as 0.1cm and [plus] 0.1cm or 1/10 cm and [plus] 1/10 cm'* to convert millimeters to centimeters. While introducing the hundredths, Nandini extended the number line with centimeters and millimeters to show the relation between meter and centimeters.

Nandini's decision to spend the whole lesson on the question of *'one division after one'* created a space for students to explore the relations between different units, thus moving beyond the procedural understanding of conversion. Public thinking in the class around the student's question led to elicitation of different representations of the same measure and justifying the relation between them. The flexibility in naming a measure using different units helped students to see the different representations of the measure. Further, Nandini's insistence on seeking for reasons, made students justify the equivalence of numerical representations using equal divisions on the ruler, decomposition of a number, and relation between place values. We conjecture that Nandini's knowledge of equal partitioning, iteration of a sub-unit to form a bigger unit, and use of a linear representation to discuss relations between units; might have helped her in uncovering the mathematical potential of the student's question. Revoicing the student's question while doing hundredths also reveals that Nandini wanted students to focus on the equal divisions and the number of divisions to link the measure meaning with the relation of tenths and hundredths. The use of consistent representation for tenths and hundredths supported students' reasoning in moving flexibly from one measurement unit to another.

CONCLUSIONS & DISCUSSION

The pattern of Nandini's responses to students reflecting an awareness of subtle aspects of the mathematical concept is also seen in other paired episodes beyond those described above. The 'tasks of teaching' involving choice of appropriate representations and their coherent use, offering reasons for equivalence of two representations and providing the tools (using representations, previously known ideas, etc.), bringing students' attention to the key mathematical ideas are in action here. The depth in Nandini's knowledge of the use of measurement as a context for learning decimal fractions supported her in unpacking the mathematical potential in students' responses. In other episodes of Nandini's teaching we have observed that her decisions are guided by anticipating potential students' difficulties and their sources, and leveraging knowledge of connections between whole numbers, fractions, and decimal numbers. We conjecture that ways in which SCK and PCK interact while the teacher is teaching is complex yet intricately related. The change in classroom practice, however, is also linked with teacher's beliefs about the student's capabilities, availability of resources (tools like textbook, research, support from peers, etc.), and the changing goals of 'what should be taught' within each topic. The change in Nandini's teaching is a complex interplay of several of these factors.

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